

## Leverage Cycles and the Anxious Economy

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*We provide a pricing theory for emerging asset classes, like emerging markets, that are not yet mature enough to be attractive to the general public. We show how leverage cycles can cause contagion, flight to collateral, and issuance rationing in a frequently recurring phase we call the anxious economy. Our model provides an explanation for the volatile access of emerging economies to international financial markets, and for three stylized facts we identify in emerging markets and high yield data since the late 1990s. Our analytical framework is a general equilibrium model with heterogeneous agents, incomplete markets, and endogenous collateral, plus an extension encompassing adverse selection. (JEL D53, G12, G14, G15)*

Since the 1990s, emerging markets have become increasingly integrated into global financial markets, becoming an asset class. Contrary to what was widely predicted by policymakers and economic theorists, however, these changes have not translated into better consumption smoothing opportunities for emerging economies. Their access to international markets has turned out to be very volatile, with frequent periods of market closures. Even worse, as we will show, emerging economies with sound fundamentals are the ones that issue less debt during these closures.

The goal of this paper is to present a theory of asset pricing that will shed light on the problems of *emerging assets* (like emerging markets) that are not yet mature enough to be attractive to the general public. Their marginal buyers are liquidity constrained investors with small wealth relative to the whole economy, who are also marginal buyers of other risky assets. We will use our theory to argue that the periodic problems faced by emerging asset classes are sometimes symptoms of what we call a global *anxious economy* rather than of their own fundamental weaknesses.

We distinguish three different conditions of financial markets: the normal economy, when the liquidity wedge is small and leverage is high; the anxious economy, when the liquidity wedge is big and leverage is curtailed, and the general public is anxiously selling risky assets to more confident natural buyers; and, finally, the crisis or panicked economy, when many formerly leveraged natural buyers are forced to liquidate or sell off their positions to a reluctant public, often going bankrupt in the process. A recent but growing literature on leverage and financial markets has concentrated on crises or panicked economies. We concentrate on the anxious economy (a much more frequent phenomenon) and provide an explanation with testable implications for (1) contagion, (2) flight to collateral, and (3) issuance rationing. Our theory provides a rationale

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for three stylized facts in emerging markets that we describe below, and perhaps also explains some price behavior of other “emerging asset” classes like the US subprime mortgage market.

In Section II we look at issuance and spread behavior of emerging market and US high yield bonds during the six-year period 1997–2002, which includes the fixed income liquidity crisis of 1997–1998. This crisis lasted for a few months, or about 4 percent of the sample period. Our estimates show, however, that during 20 percent of this period, primary markets for emerging market bonds were closed. Traditionally, periods of abnormally low access have been explained by weak emerging market fundamentals. This paper will argue that closures can often be a symptom of an anxious global economy. We will provide a theory for how shocks in other globally traded sectors like US high yield bonds can be transmitted to emerging markets even during less dramatic times than crises like the one in 1998.<sup>1</sup>

We describe three stylized facts in our data: (1) Emerging market and high yield bonds show positive spread correlation of 33 percent, even though their payoffs would seem to be uncorrelated. In particular, during emerging market closures there is an increase in spreads and volatility for both assets. (2) Although emerging market spreads increase during closures, the behavior across the credit spectrum is not the same: high-rated emerging market spreads increase less than low-rated emerging market spreads. (3) During closures the drop in issuance is not uniform either: high-rated emerging market issuance drops more than low-rated emerging market issuance. Issuance from emerging countries with sound fundamentals suffers more, even though high-rated spreads change much less.

In Section III we introduce our notion of the anxious economy. This is the state when bad news lowers expected payoffs somewhere in the global economy (say in high yield), increases the expected volatility of ultimate high yield payoffs, and creates more disagreement about high yield, but gives no information about emerging market payoffs. A critical element of our story is that bad news increases not only uncertainty, but also heterogeneity. When the probability of default is low, there cannot be much difference in opinion. Bad news raises the probability of default and also the scope for disagreement. Investors who were relatively more pessimistic before become much more pessimistic afterward. One might think of the anxious economy as a stage that is frequently attained after bad news, and that occasionally devolves into a sell-off if the news grows much worse, but which often (indeed usually) reverts to normal times. After a wave of bad news that lowers prices, investors must decide whether to cut their losses and sell, or to invest more at bargain prices. This choice is sometimes described on Wall Street as whether to catch a falling knife.

For simplicity, we suppose agents are divided into a small group of optimists, representing the natural buyers of the assets, and a large group of pessimists, representing the general public. Both groups are completely rational, forward looking, and expected utility maximizers, but with different priors. Heterogeneity and market incompleteness are important because then the valuation of an asset can depend critically on what a potentially small segment of the economy thinks of it. Even if the asset is small relative to the size of the whole economy, it might be significant relative to the wealth of the segment of the population most inclined to hold it. If markets were complete, then in equilibrium everyone on the margin would be equally inclined to hold every asset. But with incomplete markets, it may well happen that assets are entirely held by small segments of the population.

In this context, the paper presents a series of numerical simulations to answer the three questions raised by our stylized facts: (1) If bad news affects only one sector, say high yield, will asset

<sup>1</sup> Recent empirical evidence also points to the role of global factors, as in Guillermo Calvo, Alejandro Izquierdo, and Luis-Fernando Mejía (2004) and Fostel and Graciela Laura Kaminsky (forthcoming).

prices in sectors with independent payoffs like emerging markets be affected?<sup>2</sup> In other words, is contagion possible in equilibrium? (2) Why is the fall in prices of bonds for which there is no information not uniform? (3) Why is the fall in issuance of these bonds not uniform?

We show in Section III that when the economy is reducible to a representative agent, none of these things can happen. We also show that if the economy has heterogeneous investors but complete markets, and if optimists' wealth is small relative to the whole economy, then again (1), (2), and (3) will not occur.

At the end of Section III, we show that in an economy with heterogeneous investors and incomplete markets (that eliminate borrowing and short sales), it is possible to get contagion without leverage. In the anxious economy, emerging market bonds will fall in value in tandem with the high yield bonds, even though there is no new information about them. This fall derives from a *portfolio effect* and a *consumption effect*. The consumption effect arises when consumption today goes down, lowering the relative marginal utility of all assets promising future payoffs. The portfolio effect refers to the differential dependence of portfolio holdings on news. After bad news, pessimistic investors abandon high yield, and optimists take advantage of the lower prices to increase their investments in high yield. When the optimists increase their investment in high yield, they must withdraw money from somewhere else, like emerging markets and consumption. This causes the price of emerging market bonds to fall. Thus the portfolio effect gives a preliminary answer to question (1) about contagion, but it does not explain why the fall in prices (2) or issuance (3) should be nonuniform.

The portfolio effect is compatible with the recent evolution of the emerging market investor base. Emerging market bonds are still not a mature enough asset class to become attractive to the general public (the pessimists), and at the same time the marginal buyers of these assets have become crossover investors willing to move to other asset classes like high yield. The proportion of crossover investors has steadily increased. In 1996 it was approximately 15 percent, and by 2002 it accounted for more than 40 percent. Before 1997 there seems to be little correlation between high yield and emerging market spreads, but after 1999 this correlation becomes quite significant.

We define the *liquidity wedge* as the spread between the interest rate optimists would be willing to pay and the rate pessimists would be willing to take. As we shall see, the liquidity wedge is a useful way of understanding asset prices. When the liquidity wedge increases, the optimists discount the future by a bigger number, and all asset prices for which they are the marginal buyers fall. The liquidity wedge increases because the disagreement between optimists and pessimists about high yield grows, increasing the desire of optimists to get their hands on more money to take advantage of the high yield buying opportunity. The portfolio and consumption effects create a *liquidity wedge cycle*: as the real economy moves back and forth between the normal and the anxious stage, the liquidity wedge ebbs and flows.

A popular story puts the blame for contagion on leverage. Leverage (say in high yield) causes bigger losses after bad news, which causes leveraged investors to sell other assets (like emerging markets), which causes contagion. This story implicitly relies on incomplete markets (otherwise leverage is irrelevant) and on heterogeneous agents (since there must be borrowers and lenders to have leverage). The popular story is a sell-off story during panicked economies. The most optimistic buyers are forced to sell off their high yield assets, and more assets besides, holding less of high yield after the bad news than before.

<sup>2</sup> This is a pressing problem not only for emerging markets. In 2007 the subprime mortgage market suffered losses on the order of \$250 billion, which is tiny compared to the whole economy. Could this have a big effect on other asset prices?

In the popular story about crises there are usually massive defaults and bankruptcies caused by failures to meet margin calls. But these events are rare, happening once or twice a decade. Our data describe events with 10 to 20 times the frequency, happening roughly twice a year. Moreover, asset trades in the anxious stage move in exactly the opposite direction from the crisis stage. In the anxious economy it is the public that is selling in the bad news sector, and the most optimistic investors who are buying. To explain our data on emerging market closures, we tell a story that places liquidity and leverage on center stage, but that does not have the extreme behavior of the sell-off.

In order to understand the role of leverage in the anxious economy, in Section IV we introduce our model of general equilibrium with incomplete markets and collateral. Agents are allowed to borrow money only if they can put up enough collateral to guarantee delivery. Assets in our model play a dual role: they are investment opportunities, but they can also be used as collateral to gain access to cash. The *collateral capacity* of an asset is the level of promises that can be made using the asset as collateral. This is an endogenous variable that depends on expectations about the distribution of future asset prices. Together with the interest rate, the collateral capacity determines an asset's *borrowing capacity*, which is the amount of money that can be borrowed using the asset as collateral. The *loan to value* (LTV) of an asset is the *ratio* of the asset's borrowing capacity to its price. The *haircut* or *margin* of an asset is the shortfall of its LTV from 100 percent—in other words, the fraction of the price that must be paid in cash. The maximal *leverage* of an asset is the inverse of its margin. The leverage in the system, like the other ratios just mentioned, is determined by supply and demand; it is not fixed exogenously.

We derive a pricing lemma which shows that the price of an asset can always be decomposed as the sum of its *payoff value* and its *collateral value* to any agent who holds it. Ownership of an asset not only gives the holder the right to receive future payments (reflected in the payoff value), but also enables the holder to use it as collateral to borrow more money. The collateral value reflects the asset's marginal contribution to an agent's liquidity. The collateral value of an asset to an agent is the product of the asset's collateral capacity, and a term depending on how valuable liquidity is to the agent as measured by the liquidity wedge, discounted by the interest rate. The collateral value of an asset rises as the liquidity wedge rises. But the payoff value of an asset falls as the liquidity wedge rises. In general, the effect of a change in the liquidity wedge on payoff values is bigger than the effect on collateral values (holding collateral capacities constant), so an increase in the liquidity wedge tends to lower asset prices.

Our model shows that leverage tends to increase asset values for two reasons. First, it permits more borrowing, and hence tends to lower the liquidity wedge, which in turn raises payoff values more than it reduces collateral values. Second, by increasing collateral capacities it also tends to increase collateral values, mitigating the fall in collateral values from the declining liquidity wedge.

Equilibrium leverage is not constant, either across states or across assets. We find a *leverage cycle* in equilibrium: leverage rises in the normal state and falls in the anxious stage. Even if asset prices did not change, borrowing capacities would fluctuate because the LTV endogenously fluctuates. Indeed, the change in borrowing capacity caused by changes in LTV or leverage is much bigger than that caused by changes in asset prices.

The underlying dynamic of the anxious economy—fluctuating uncertainty and disagreement—simultaneously creates the leverage cycle and the liquidity wedge cycle; that is why they run in parallel. Since leverage affects the liquidity wedge, the leverage cycle amplifies the liquidity wedge cycle. So what does collateral, and the possibility of leverage, add to the liquidity wedge cycle already discussed? It generates a bigger price crash, not due to *asset undervaluation* during anxious times, but due to *asset overvaluation* during normal times. This may lead the press to talk about asset price bubbles. The liquidity wedge cycle reinforced by the leverage cycle provides our full answer to question (1) and rationalizes Stylized Fact 1.

While leverage was not necessary for contagion, it plays the shinning role in our answer to question (2). Traditionally the deterioration in price of low-quality assets is explained in terms of “flight to quality,” which in our model corresponds to movements in payoff values. Our model provides a different and complementary channel originating exclusively from collateral, and hence liquidity, considerations. We find that in the anxious economy asset prices generally fall, but collateral values often rise, and so assets with higher collateral values fall less. We call this phenomenon *flight to collateral*.

Flight to collateral arises in equilibrium when: (a) the liquidity wedge is high and (b) the dispersion of margins between assets is high. The key is that different assets experience different leverage cycles, because they all have their own endogenous margins (collateral capacities) in equilibrium. The liquidity wedge has a common effect on all asset prices, but the collateral values also depend on the idiosyncratic margins. The differentiated behavior in collateral values explains the differential fall in prices. The good emerging market asset has a significantly higher collateral value than the bad emerging market asset during the anxious economy. During a flight to collateral episode, investors would rather *buy* those assets that enable them to borrow money more easily. The other side of the coin is that investors who need to raise cash get more by *selling* those assets on which they did not borrow money because the sales revenues net of loan repayments are higher.

The model provides the following testable implication. We show that even when two assets have the same information volatility, margins during normal times will be different and can predict which assets are the ones that will suffer more during future flight to collateral episodes. Our second result rationalizes Stylized Fact 2, since low-rated emerging market bonds exhibit higher margins than high-rated emerging market bonds.

To address question (3) about why the fall in issuance during closures is not uniform, Section V extends our first model to encompass the supply of emerging market assets as well as asymmetric information between countries and investors. We show that flight to collateral combined with asymmetric information between investors and countries leads to issuance rationing. During episodes of global anxiety, the big liquidity wedge creates high collateral values and high collateral value differentials. “Good” type country assets are better collateral and suddenly become worth much more than “bad” type country assets, at least to people who recognize the difference. When investors cannot perfectly observe these types, only a drastic drop in good type issuance removes the incentive of bad types to mimic good types, maintaining the separating equilibrium. In a world with no informational noise, spillovers from other markets and flight to collateral may even help “good” issuance. With informational noise between countries and investors, however, good quality issuance paradoxically suffers more, rationalizing Stylized Fact 3.

### I. Relation with the Literature

The starting points for our analysis are Geanakoplos (2003) and Fostel (2005). The first paper described what we now call the leverage cycle, focusing on the crisis stage. The second paper extended the leverage cycle to an economy with multiple assets and introduced what we now call the anxious economy.

Our first model of collateral equilibrium follows the tradition of collateral general equilibrium introduced by Geanakoplos (1997), and in more general form, by Geanakoplos and William R. Zame (1998). Geanakoplos (1997) and especially Geanakoplos (2003) introduced the idea of endogenous margins or equilibrium leverage. Geanakoplos (2003) also identified increasing volatility and increasing disagreement as causes of increased margins, and hence of the leverage cycle, in the same way we do here. Although in these papers it is clear that the price of a



collateralizable asset is not equal to its payoff value, the explicit decomposition we give here in the pricing lemma of asset price into payoff value and collateral value is new.

Our second model of adverse selection with endogenous collateral has as its root the classic paper on signalling in insurance markets by Michael Rothschild and Joseph Stiglitz (1976). That paper mixes competitive equilibrium with Nash equilibrium, creating difficulties for the existence of equilibrium. Prakeep K. Dubey and Geanakoplos (2002) recast the insurance model in a perfectly competitive framework with pooling, proving both existence and uniqueness of equilibrium. We extend the Dubey and Geanakoplos (2002) approach to a much more complicated model with endogenous leverage.

There is a long and important tradition of work on credit constraints in macroeconomics. Most relevant for us are the papers by Nobuhiro Kiyotaki and John Moore (1997), Ben Bernanke, Mark Gertler, and Simon Gilchrist (1996), and Ricardo J. Caballero and Arvind Krishnamurthy (2001). All these papers point out that when margins are exogenously fixed, the amount of borrowing goes up proportionately with the prices of collateralizable assets, giving a positive feedback. But they did not observe that it is possible to make the margins themselves (and hence the degree of leverage) endogenous. Following Geanakoplos (2003), we see in this paper that the endogenous change in the leverage and LTV of a collateralizable asset can generate a much bigger feedback than the change in the price of the asset alone.<sup>3</sup>

Our second model is related to an increasing literature that tries to model asymmetric information within general equilibrium, like Douglas Gale (1992), Alberto Bisin and Piero Gottardi (2006), and Aldo Rustichini and Paolo Siconolfi (forthcoming). The main contribution of this paper to the literature is to model asymmetric information in a general equilibrium model with incomplete markets and endogenous credit constraints; to the best of our knowledge such blending has not been done before. Our assumption of asymmetric information between investors and countries is related to several papers in the sovereign debt literature, as in Jonathan Eaton (1996), Laura Alfaro and Fabio Kanuczuk (2005), and Luis Catão, Fostel, and Sandeep Kapur (forthcoming).

Our paper is related to an extensive literature on contagion. Despite the range of different approaches, there are mainly three different kinds of models. The first blends financial theories with macroeconomic techniques, and seeks international transmission channels associated with macroeconomic variables. Examples of this approach are Ilan Goldfajn and Rodrigo Valdes (1997), Pierre Agenor and Joshua Aizenman (1998), Giancarlo Corsetti, Paolo Pesenti, and Nouriel Roubini (1999), and Anna Pavlova and Roberto Rigobon (forthcoming). The second kind models contagion as information transmission. In this case the fundamentals of assets are assumed to be correlated. When one asset declines in price because of noise trading, rational traders reduce the prices of all assets since they are unable to distinguish declines due to fundamentals from declines due to noise trading. Examples of this approach are Mervyn King and Sushil Wadhvani (1990), Calvo (1999), Calvo and Enrique Mendoza (2000), Marco Cipriani and Antonio Guarino (forthcoming), and Laura Kodres and Matthew Pritsker (2002). Finally, there are theories that model contagion through wealth effects, as in Albert Kyle and Wei Xiong (2001). When some key financial actors suffer losses, they liquidate positions in several markets, and this sell-off generates price comovement. Our paper shares with the last two approaches a focus exclusively on contagion as a financial market phenomenon. Our main contribution to this literature consists in showing how leverage cycles can produce contagion in less extreme but more frequent market conditions: the anxious economy. The leverage cycle causes contagion through different trade patterns and price behavior from those observed during acute crises.

<sup>3</sup> In one of our simulations, the LTV drops from 87 percent to 13 percent (equivalently, leverage falls from 7.8 to 1.2), while the asset price declines only 13 percent.

Flight to collateral is related to what other papers have called flight to liquidity. Flight to liquidity was discussed by Dimitri Vayanos (2004) in a model where an asset's liquidity is defined by its exogenously given transaction cost. In Markus Brunnermeier and Lasse Pedersen (forthcoming), market liquidity is the gap between fundamental value and the transaction price. They show how this market liquidity interacts with funding liquidity (given by trader's capital and margin requirements) generating flight to liquidity. In our paper we model an asset's liquidity as its capacity as collateral to raise cash. Hence, our flight to collateral arises from different leverage cycles in equilibrium and their interaction with the liquidity wedge cycle.

On the empirical side, our paper presents three stylized facts. While the first two facts regarding spreads confirm what has also been found by other empirical studies (see for example Martin Gonzalez and Eduardo Levy Yeyati 2005), the third stylized fact regarding differential issuance during closures is new, to the best of our knowledge.

Finally, our model is related to a vast literature that explains financial crises, sudden stops, and lack of market access in emerging market economies. The sovereign debt literature, as in Jeremy Bulow and Kenneth Rogoff (1989), stresses moral hazard and reputation issues. The three "generations" of models of currency crises explain reversals in capital flows by pointing to fiscal and monetary causes as in Paul Krugman (1979), to unemployment and overall loss of competitiveness as in Maurice Obstfeld (1994), and to banking fragility and overall excesses in financial markets as in Kaminsky and Carmen Reinhart (1999) and Roberto Chang and Andres Velasco (2001). Others explore the role of credit frictions to explain sudden stops, as in Calvo (1998) and Mendoza (2004). Others focus on balance sheet effects, as in Krugman (1999), Philippe Aghion, Philippe Bacchetta, and Arindam Banerjee (2006), Martin Schneider and Aaron Tornell (2004), and finally on the interaction of financial and goods markets, as in Philippe Martin and Hélène Rey (2006).

## II. Stylized Facts

Following Fostel (2005), we look at emerging markets' issuance of dollar-denominated sovereign bonds covering the period 1997–2002. The data we use are obtained by Dealogic, which compiles daily information on issuance at the security level. We define a *primary market closure*<sup>4</sup> as a period of three consecutive weeks or more during which the weekly primary issuance over all emerging markets is less than 40 percent of the period's trend. As shown in Table 1, market closures are not rare events. During this period, there were 13 market closures, which implies that 20.29 percent of the time primary markets of emerging market bonds were closed. While some of the closures seem associated with events in emerging countries, others seem to correspond with events in mature economies. The events we wish to explain are thus not once-in-a-decade crises, like the fixed income crash of 1997–1998, but recurring episodes that happen on the order of once or twice a year. Presumably, they are not characterized by drastic changes in consumption.

During the same period, we look at the secondary markets of emerging markets and US high yield bonds. We use daily data on spreads from the JPMorgan index EMBI+ for emerging markets and the Merrill Lynch index for US high yield. Data for emerging market spreads disaggregated by credit ratings are available at weekly frequency from Merrill Lynch indexes.<sup>5</sup>

<sup>4</sup> We follow the IMF's methodology to calculate closures, as in its Global Financial Stability Report (GFSR) (March 2003).

<sup>5</sup> Although spreads at issuance, which reflect the actual cost of capital, may be the most relevant for the issuer, portfolio managers arguably follow spreads in secondary markets more closely. Also, these spreads available at higher frequency may reflect subtle changes in global investing conditions more accurately than lower frequency data.

TABLE 1—PRIMARY MARKET CLOSURES FOR EMERGING MARKET BONDS, 1997–2002

| Closure | Year | Date        | Duration (weeks) | Associated event           |
|---------|------|-------------|------------------|----------------------------|
| 1       | 1997 | 03/17–04/06 | 3                | Thailand turmoil           |
| 2       | 1997 | 08/18–09/07 | 3                | Thailand devaluation       |
| 3       | 1997 | 10/27–12/07 | 6                | Korea crisis               |
| 4       | 1998 | 08/03–10/26 | 12               | Russia default and LTCM    |
| 5       | 1999 | 01/01–01/31 | 4                | Brazil devaluation         |
| 6       | 1999 | 07/12–08/02 | 3                |                            |
| 7       | 1999 | 08/16–09/05 | 3                |                            |
| 8       | 2000 | 04/03–05/01 | 4                | US interest rate anxieties |
| 9       | 2000 | 09/25–10/30 | 5                | US stock market crash      |
| 10      | 2001 | 08/20–09/10 | 3                | US recession concerns      |
| 11      | 2002 | 04/29–06/17 | 7                | Brazil turmoil             |
| 12      | 2002 | 08/05–09/02 | 4                | US stock market            |
| 13      | 2002 | 09/23–10/14 | 3                |                            |

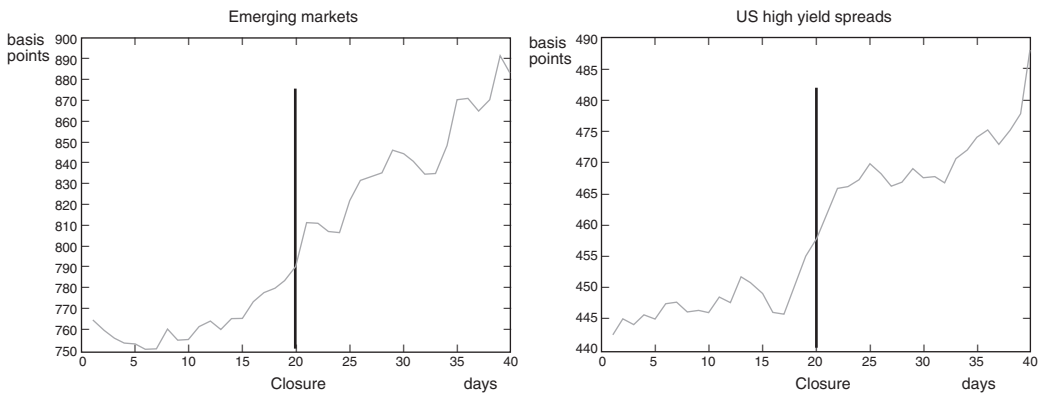


FIGURE 1. AVERAGE SPREADS AROUND CLOSURES

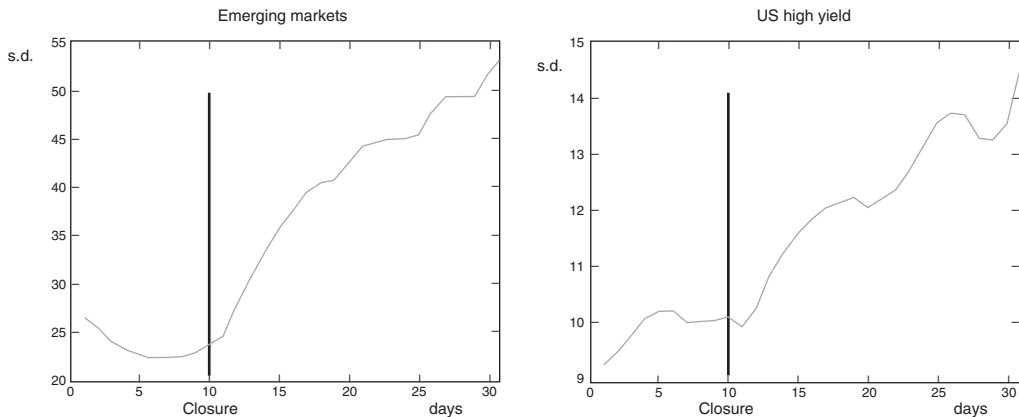


FIGURE 2. AVERAGE SPREAD VOLATILITY AROUND CLOSURES

*Stylized Fact 1: Emerging Markets and US High Yield Spread Correlation.*—The average correlation during the period is 0.33. Figure 1 shows average spread behavior for both assets from 20 days before to 20 days after the beginning of a typical closure. The increasing behavior around closures is also true for the 20-day rolling spread volatility as shown in Figure 2. This increasing pattern is robust across all closures in the sample and to different rolling windows specifications.



*Stylized Fact 2: Credit Rating and Emerging Markets Spreads.*—Although emerging market spreads increase around market closures, the behavior across the credit spectrum within the asset class is not uniform: high-rated emerging market spreads increase less than low-rated emerging markets spreads.<sup>6</sup>

Figure 3 shows the average weekly percentage change in spreads around closures for different emerging market ratings. On average, low-rated spreads increase more than high-rated spreads, and this behavior is robust across closures as well.

*Stylized Fact 3: Credit Rating and Emerging Market Primary Issuance.*—During primary market closures, the drop in issuance is not uniform across the credit spectrum: high-rated emerging market issuance drops more than low-rated emerging market issuance. While high-rated issuance accounts for 23 percent during normal times, it accounts for only 12 percent during closures. Hence during closures, emerging economies with sound fundamentals seem to suffer more (issue less). One may argue that we should expect this behavior since precisely those good fundamentals allow countries to tap better alternative financial opportunities during bad times. However, this drastic reduction in issuance is puzzling when considered jointly with the behavior in spreads described before: high-rated issuance decreases more than low-rated issuance despite the fact that high-rated spreads increase less than low-rated spreads.

Finally, given the ad hoc nature of the definition of market closures, we conduct a robustness check for different thresholds and trend specifications. All three stylized facts remain remarkably robust to all these different specifications.<sup>7</sup>

### III. The Problem

#### A. The Anxious Economy

We introduce the theoretical problem motivated by the empirical section through a simple example described in Figure 4. Consider a world with a single perishable consumption good, a long-lived high yield asset  $H$ , and two long-lived emerging market assets  $E$  of differing quality,  $E^G$  and  $E^B$  (good and bad type of emerging markets). Asset payoffs are denominated in units of the single consumption good. These payoffs come in the terminal nodes, and are uncertain. Payoffs for  $H$ ,  $E^G$ , and  $E^B$ , respectively, are listed at the right of Figure 4.

Agents have riskless initial endowments  $e$  of the consumption good at each node. While agents are endowed with  $H$ , they need to buy  $E^G$  and  $E^B$  from emerging countries, which at each state enter the market and decide their issuance.

We shall suppose that news about  $H$  arrives between periods 1 and 2, and more news about  $H$  and  $E$  arrives between periods 2 and 3. Good news corresponds to up,  $U$ , and more bad news to down,  $D$ . Arriving at  $D$  makes everyone believe that  $H$  is less likely to be productive, but gives no information about  $E^G$  and  $E^B$ . After  $U$  (which occurs with probability  $q$ ), the output of  $H$  is 1 for certain, but after  $D$  the output of  $H$  can be either 1, with probability  $q$ , or  $H < 1$ , with probability  $1 - q$ . The output of  $E^G$  ( $E^B$ ) is either 1, with probability  $q$ , or  $G$  ( $B$ ), with probability  $1 - q$ , irrespective of whether  $U$  or  $D$  is reached and independent from the output of  $H$ .  $H$ ,  $G$ , and  $B$  can be interpreted as recovery values in the case of asset default and are such that  $H < 1$ ,  $B \leq G < 1$ .

<sup>6</sup> By low-rated we mean all subinvestment grade bonds, i.e., everything below or equal to BB as defined by Standard & Poor's.

<sup>7</sup> Results are available from the authors upon request.

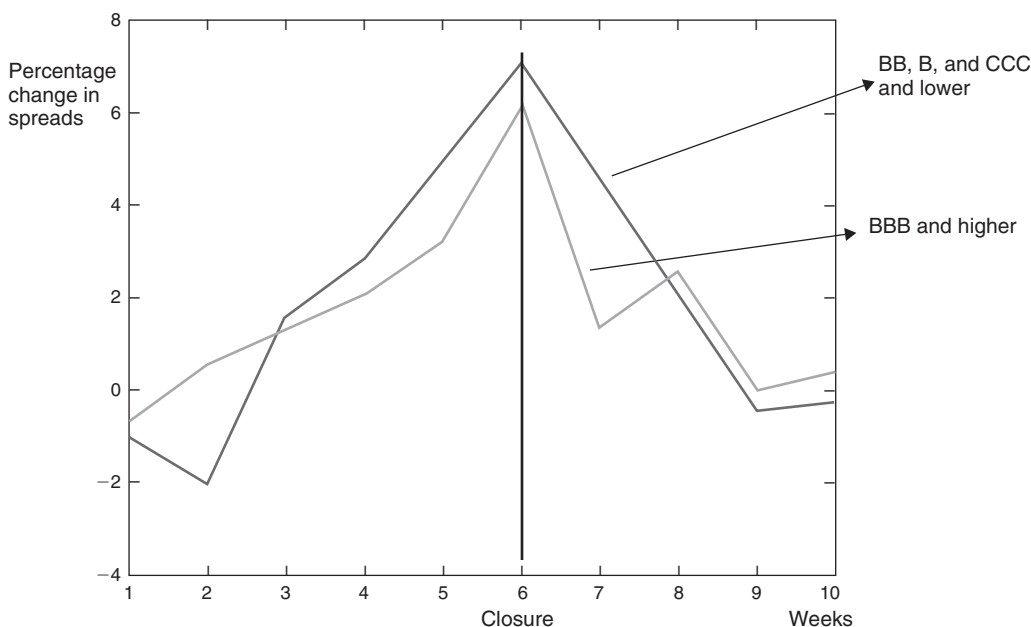


FIGURE 3. AVERAGE PERCENTAGE CHANGE IN EMERGING MARKET SPREADS BY CREDIT RATINGS AROUND CLOSURES

At  $U$  the uncertainty about  $H$  is resolved, but at  $D$  it becomes greater than ever. This stands in sharp contrast with traditional financial models, where asset values are modeled by Brownian motions with constant volatilities.

We call state  $D$  the *anxious economy*. This is the state occurring just after bad news lowers expected payoffs in high yield (our proxy for the global economy), increases the expected volatility of ultimate high yield payoffs, and creates more disagreement about high yield, but gives no information about emerging market payoffs. State  $D$  will not turn out to be a crisis situation because agents get a new infusion of endowments  $e$ .

In discussing asset price changes, we must keep in mind how much news is arriving about payoff values. We would expect asset prices to be more volatile if there were a lot of news about their own payoff, and to be less volatile or even flat if there were no news. In our setup there is an acceleration of news over time, and eventually more news about  $E^B$  than about  $E^G$ . There are situations when this kind of uncertainty is natural, for example, if everyone can see that a day is approaching when some basic uncertainty is going to be resolved.<sup>8</sup>

To be precise, for each asset  $A$  and each node  $s$ , let us define  $\mathcal{E}_s(A)$  as the expected terminal delivery of  $A$  conditional on having reached  $s$ . Similarly, define the *informational volatility* at  $s$ ,  $V_s(A)$ , as the standard deviation of  $\mathcal{E}_\alpha(A)$  over all immediate successors  $\alpha$  of  $s$ .<sup>9</sup> Then, at  $D$  the expectation of  $H$  drops,  $\mathcal{E}_D(H) < \mathcal{E}_1(H) < \mathcal{E}_U(H)$ , and the volatility rises,  $V_D(H) > V_1(H) >$

<sup>8</sup> At the present time everyone can see that in 2009 the subprime mortgages from the bad 2006 vintage will reset and then it will be revealed how bad defaults are.

<sup>9</sup> Then,  $\mathcal{E}_1(H) = (1 - (1 - q)^2)1 + (1 - q)^2H$ ,  $\mathcal{E}_U(H) = 1$  and  $\mathcal{E}_D(H) = q1 + (1 - q)H$ . Thus,  $V_1(H) = (1 - H)\sqrt{(1 - (1 - q)^2)(1 - q)^2}$ ,  $V_U(H) = 0$  and  $V_D(H) = (1 - H)\sqrt{q(1 - q)}$ . On the other hand,  $\mathcal{E}_1(E^G) = q1 + (1 - q)G = \mathcal{E}_U(E^G) = \mathcal{E}_D(E^G)$ . Thus,  $V_1(E^G) = 0$ ,  $V_U(E^G) = V_D(E^G) = (1 - G)\sqrt{q(1 - q)}$ . Similarly,  $\mathcal{E}_1(E^B) = q1 + (1 - q)B = \mathcal{E}_U(E^B) = \mathcal{E}_D(E^B)$ . Thus,  $V_1(E^B) = 0$ ,  $V_U(E^B) = V_D(E^B) = (1 - B)\sqrt{q(1 - q)}$ .

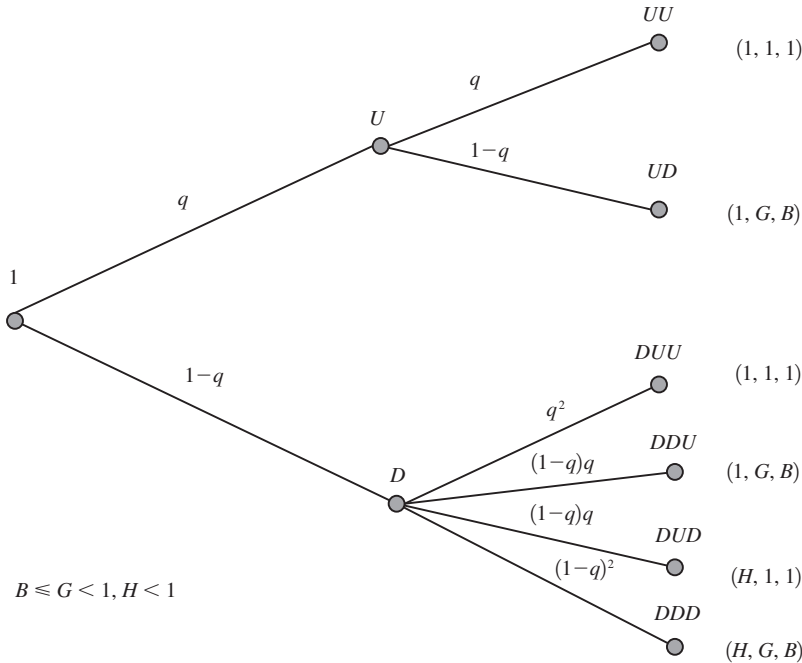


FIGURE 4

$V_U(H)$ . On the other hand, since there is no information about the payoffs of  $E^G$  and  $E^B$  between periods 1 and 2,  $V_1(E^G) = V_1(E^B) = 0$ . Eventually there is more news about  $E^B$  than  $E^G$ , so  $0 < V_U(E^G) = V_D(E^G) < V_U(E^B) = V_D(E^B)$ , provided  $B < G < 1$ .

Naturally the price of  $H$  falls from 1 to  $D$  and is lower at  $D$  than at  $U$  since the bad news lowers its expected payoff. However, the expected payoff of  $E^G$  (and  $E^B$ ) is exactly the same at  $U$  and at  $D$ , as is its information volatility. So we ask:

1. Why should the prices of  $E^G$  and  $E^B$  fall from 1 to  $D$  and be lower at  $D$  than at  $U$  (even without a shock to them)? We will refer to this problem as *Contagion*.
2. Why should the price of  $E^B$  fall more than the price of  $E^G$  from 1 to  $D$ ? And why should the gap in prices between  $U$  and  $D$  be bigger for  $E^B$  than for  $E^G$ ? We will refer to this problem as *Differential Contagion*. Moreover, is there a market signal at time 1 that can predict which asset will perform worse at  $D$ ?
3. Why should emerging market issuance fall from 1 to  $D$ , but, more importantly, why should the issuance of  $E^G$  fall more than the issuance of  $E^B$ ? And why should the gap in issuance between  $U$  and  $D$  be bigger for  $E^G$  than for  $E^B$ ? We will refer to this problem as *Issuance Rationing*.<sup>10</sup>

Answers to questions 1, 2, and 3 will help rationalize Stylized Facts 1, 2, and 3, respectively. The first model in Section IV will focus on contagion and differential contagion, while the sec-

<sup>10</sup> Though what we see in the data corresponds to movements from 1 to  $D$ , from a theoretical point of view it makes sense to compare with the counterfactual state  $U$  as well.

ond model in Section V will focus on issuance rationing. Hence, until Section V we will assume a fixed supply of emerging market assets.

Before introducing the first model, let us go back to our example and attempt to gain intuition about what is involved in solving the first two problems within standard models.

### B. Representative Agent

For a moment, let us abstract from different types of emerging market assets and consider only two assets,  $E$  (Emerging Market) and  $H$  (High Yield), with independent payoffs as discussed before.<sup>11</sup> Intuitively, since  $E$  and  $H$  are independent assets, one would expect uncorrelated price behavior in equilibrium. And, in fact, this intuition is correct in certain cases, as we will discuss now.

Consider an economy with a representative investor with logarithmic utility who does not discount the future. Simulation 1 calculates equilibrium prices for the following parameter values:<sup>12</sup> the recovery values are  $E = 0.1$  and  $H = 0.2$ , initial endowments are  $e = 2020$  in every node, beliefs are  $q = 0.9$ , and finally the agent is endowed with two units of  $H$  and two units of  $E$  in period 1. Panel A of Table 2 shows that the price of  $H$  falls at  $D$  since its expected output is lower. But the equilibrium price of  $E$  is slightly higher at  $D$  than at  $U$ , so  $E$  and  $H$  are actually slightly negatively correlated. There is no contagion. The reason for this is very simple: at  $D$ , future consumption is lower than at  $U$  since  $H$  is less productive, so the marginal utility for future output (such as that from  $E$ ) is slightly higher.

### C. Heterogeneous Agents and Complete Markets

Let us extend the previous model to allow for heterogeneous agents. Agents will differ in beliefs and wealth. There are “optimists” who assign probability  $q^O = 0.9$ , and “pessimists” who assign probability  $q^P = 0.5$ , to good news about  $H$  and  $E$ . Both agents think  $H$  and  $E$  are uncorrelated, but the optimists view both assets more favorably. At 1, optimists think  $H$  will pay fully with probability  $1 - (1 - q^O)^2 = 0.99$ , while pessimists attach only probability  $1 - (1 - q^P)^2 = 0.75$  to the same event. At  $D$  their opinions about  $H$  fully paying diverge even more,  $q^O = 0.9 > q^P = 0.5$ . This growing dispersion of beliefs after bad news is not universal, but is plausible in some cases and will be important to our results. Initial endowments are  $e^O = 20$  and  $e^P = 2,000$  for optimists and pessimists, respectively, in all states. Each type of investor owns one unit of each asset at the beginning. The rest of the parameters are as in Simulation 1.

Suppose for now that markets are complete in the sense that all Arrow securities are present. Panel B of Table 2 shows that prices exhibit only a tiny degree of contagion. The reason for any contagion is that with complete markets, agents are able to transfer wealth to the states they think are relatively more likely. Therefore, at  $U$ , prices reflect the optimist’s preferences slightly more than they do at  $D$  (and hence may be slightly higher than at  $D$ ). As we make pessimists richer and richer, however, this type will become close to a representative agent and *all* prices will reflect only his preferences. A small group of optimists cannot have much effect on asset prices when markets are complete; this is confirmed in Simulation 2. In the limit, contagion will completely disappear, as shown by Simulation 1. By contrast, we will see that with incomplete markets, making pessimists richer will not kill contagion; in fact, it will make contagion worse.

<sup>11</sup> Equivalently, assume that  $G = B$ , so there is no difference between emerging market assets.

<sup>12</sup> Sections IIID and IVD will discuss this choice of parameter values extensively.

TABLE 2—SIMULATIONS 1 AND 2

| Asset   | $p_1$  | $p_U$  | $p_D$  | $(p_U - p_D)/p_U$<br>% | $(p_1 - p_D)/p_1$<br>% |
|---|--------|--------|--------|------------------------|------------------------|
| <i>Panel A. Representative agent</i>                      |        |        |        |                        |                        |
| <i>E</i>  | 0.9082 | 0.9082 | 0.9083 | -0.01                  | -0.01                  |
| <i>H</i>  | 0.9901 | 0.9981 | 0.9183 | 8.00                   | 7.25                   |
| <i>Panel B. Complete markets and heterogeneous agents</i> |        |        |        |                        |                        |
| <i>E</i>  | 0.5527 | 0.5554 | 0.5499 | 1.0                    | 0.5                    |
| <i>H</i>  | 0.8007 | 0.9985 | 0.5998 | 39.9                   | 25.1                   |

#### D. Incomplete Markets and Heterogeneous Agents

*Contagion, Portfolio Effect, and Consumption Effect.*—Simulations 1 and 2 show that contagion without correlated fundamentals is not a general phenomenon. The first example illustrates the need for some kind of agent heterogeneity, while the second highlights the need for market incompleteness. In the next example we will assume both. Agents are heterogeneous. As before, they differ in beliefs and endowments, which are given by  $q^O = 0.9$ ,  $q^P = 0.5$ ,  $e^O = 20$ , and  $e^P = 2,000$ , respectively. Each type of investor starts with one unit of each asset  $E$  and  $H$  at the beginning and trades these assets thereafter.

But now markets are assumed to be incomplete. Agents can trade only the physical assets  $E$  and  $H$ , and the consumption good. Arrow securities are assumed not present and agents are not allowed to borrow or to sell short. Given that  $D$  is followed by four states, two assets are not enough to complete markets. But even at 1, markets are incomplete due to the presence of short sales constraints.<sup>13</sup>

Let us take a moment to discuss parameter values before presenting Simulation 3. As before, we assume that  $H$ 's recovery value is bigger than  $E$ 's,  $H = 0.2$ ,  $E = 0.1$ . This constitutes a realistic assumption since in general the recovery value from a domestic firm is bigger than the one from foreign countries due to the absence of international bankruptcy courts. As above, investors have logarithmic utilities and do not discount the future. We think of optimists as the class of investors who find emerging markets an attractive asset class, whereas pessimists are thought of as the "normal public" who invest in the US stock market. While the market for emerging market bonds accounted for approximately \$200 billion, the US stock market accounted for approximately \$20 trillion by the end of 2002. Hence, we have given pessimists 100 times the wealth of optimists.

Results for Simulation 3 are shown in Tables 3, 4, and 5. The first thing to notice is that asset prices are much higher in Simulation 3 than in Simulation 2. On account of the incomplete markets, the marginal buyer of the assets is the optimist, so the prices reflect his higher expectations.

However, there is a more interesting difference. In Simulation 1, the optimist was also the (only) marginal buyer, yet there was no contagion. In Simulation 3, prices for  $E$  and  $H$  rise at  $U$  and fall at  $D$ , displaying contagion. Along the path from 1 to  $D$  of bad news about  $H$ , the price of  $H$  naturally falls, declining 19 percent from 0.9 to 0.74. The price of  $E$  falls as well from 1 to  $D$ , even though there was no specific bad shock to it. It goes from 0.8 to 0.73, a decline of 8.6 percent. The difference in prices between  $U$  and  $D$  for  $H$  is 26.25 percent and for  $E$  is 15.7 percent.

Why does  $E$  fall in price in the anxious economy? First, because of a *portfolio effect*. Second, because of a *consumption effect*.

What is crucial in the portfolio effect is that optimists hold more of  $H$  after bad news than after good news about  $H$ . At  $U$ , news is so good that both types agree about  $H$  and optimists end

<sup>13</sup> Market incompleteness means that there is a node at which agents, at equilibrium prices, cannot create all the Arrow securities that span the dimension of the set of successor states.

TABLE 3—SIMULATION 3, INCOMPLETE MARKETS: PRICES

|          | 1      | <i>U</i> | <i>D</i> | $(U-D)/U$<br>% | $(1-D)/1$<br>% |
|----------|--------|----------|----------|----------------|----------------|
| $\omega$ | 0.0668 | 0.0447   | 0.2429   |                |                |
| Asset    |        |          |          |                |                |
| <i>E</i> | 0.7954 | 0.8630   | 0.7273   | 15.72          | 8.56           |
| <i>H</i> | 0.9097 | 0.9986   | 0.7364   | 26.25          | 19.05          |

TABLE 4—SIMULATION 3, INCOMPLETE MARKETS: PORTFOLIO

| Asset    | 1        |          | <i>U</i> |          | <i>D</i> |          |
|----------|----------|----------|----------|----------|----------|----------|
|          | <i>O</i> | <i>P</i> | <i>O</i> | <i>P</i> | <i>O</i> | <i>P</i> |
| <i>E</i> | 0.6624   | 1.3376   | 2        | 0        | 2        | 0        |
| <i>H</i> | 2        | 0        | 0        | 2        | 2        | 0        |

TABLE 5—SIMULATION 3, INCOMPLETE MARKETS: CONSUMPTION AND ADJUSTED PROBABILITIES

| Cons.   | 1      | <i>U</i> | <i>D</i> | <i>UU</i> | <i>UD</i> | <i>DUU</i> | <i>DDU</i> | <i>DUD</i> | <i>DDD</i> |
|---------|--------|----------|----------|-----------|-----------|------------|------------|------------|------------|
| $\mu^O$ |        | 0.892    | 0.109    | 0.892     | 0.108     | 0.798      | 0.096      | 0.095      | 0.012      |
| $x^O$   | 19.40  | 20.80    | 19.00    | 22.00     | 20.20     | 24.00      | 22.20      | 22.40      | 20.60      |
| $x^P$   | 2000.6 | 1999.2   | 2001     | 2002      | 2002      | 2000       | 2000       | 2000       | 2000       |

up holding none of it. However, at *D*, when asset volatility has gone up, the difference in opinion increases, so optimists see a special opportunity and end up holding all of *H*. Given constant wealth, they have relatively less wealth to spend on *E* and on consumption. The reduction in the demand for *E* naturally lowers its price. Equivalently, the portfolio effect generates a consumption effect: optimists' consumption goes down (by 9 percent) and their marginal utility goes up from *U* to *D*, reducing the marginal utility of *E* relative to consumption. Thus, the price of *E* mimics the price of *H*. Since the price at 1 is an average of the prices at *U* and *D*, the portfolio effect also implies that the price falls from 1 to *D*. The portfolio and consumption effects also explain why the fall of 26.25 percent in the price of *H* from *U* to *D* is bigger than the fall in its (optimistic) expected payoff of 8 percent.

Investor heterogeneity and market incompleteness are what generate the portfolio and consumption effects; without them, contagion may well disappear. Heterogeneous beliefs (at time 1) make emerging market assets less attractive to the "normal public," modeled here as pessimists, but extremely attractive to another class of investors, modeled here as optimists. Contagion becomes possible when these optimistic investors become "crossover" investors, ready to move part of their capital to high yield bonds when they see a special opportunity.

This portfolio effect is in line with important changes that have taken place in the investor base for emerging market assets in recent years: the proportion of crossover investors has steadily increased. In 1996 it was approximately 15 percent, and by 2002 it accounted for more than 40 percent.<sup>14</sup> The portfolio effect jointly with the change in investor base help to explain why the correlation between emerging markets and US high yield spreads started to become very significant after 1997.<sup>15</sup>

<sup>14</sup> See IMF, GFSR (September 2003).

<sup>15</sup> See IMF, GFSR (March 2003). Gonzalez and Levy Yeyati (2005) find the breaking point at 1999.



On the other hand, the impact of hedge funds, through their leveraged positions, on contagion has received substantial attention in both academic and official communities. Yet, while in 1998 leveraged investors such as hedge funds accounted for 30 percent of all activity in emerging markets, this share declined to 5 percent by 2002.<sup>16</sup>

Simulation 3 shows that leverage is not necessary to generate contagion; portfolio and consumption effects are enough. Since it is usual to associate contagion with leverage, we will introduce collateral, and hence leverage, in Section IV in order to understand its role in contagion. We will see, first, that leverage will reduce contagion as measured by a fall from  $U$  to  $D$ , but it will generate a bigger price crash from 1 to  $D$ . Second, the trading dynamic behind the asset price plunge from 1 to  $D$  in the anxious economy will be different from what one sees in crisis economies.

*Liquidity Wedge Cycle.*—On account of the incomplete markets, optimists cannot borrow money from pessimists. We call the spread between the interest rate optimists would be willing to pay and the rate the pessimists would be willing to take the liquidity wedge. The missing loan market creates an inefficient liquidity wedge between borrowers and lenders, leaving potential gains from trade unexploited. Table 3 shows that the liquidity wedge,  $\omega$ , increases from  $U$  to 1 to  $D$ . As we shall see in further simulations, the liquidity wedge is a useful way of understanding asset prices. When the liquidity wedge increases, the optimists discount the future by a bigger number, and asset prices fall (assuming the optimists are marginal buyers). The liquidity wedge increases because the disagreement between optimists and pessimists about  $H$  grows, increasing the desire of optimists to get their hands on more money to take advantage of the  $H$  buying opportunity (portfolio and consumption effects).<sup>17</sup> As the real economy moves back and forth between the normal and the anxious stage, the liquidity wedge ebbs and flows.

*Differential Contagion.*—Consider our example with three assets,  $H$ ,  $E^G$ , and  $E^B$ , and  $B \leq G$ . Are the portfolio and consumption effects, operating through the liquidity wedge, enough to generate not only contagion but also differential contagion across emerging market assets of differing quality in the anxious economy?

Simulation 4 calculates the equilibrium for the same parameter values as before, except for the recovery values, which are now  $H = 0.2$ ,  $G = 0.2$ , and  $B = 0.05$  (the emerging market asset  $E$  with recovery value 0.1 is replaced by a good emerging market asset with higher recovery value, 0.2, and a bad emerging market asset with a lower recovery value, 0.05). Each agent is endowed with one unit of  $H$  and 0.5 units of  $E^G$  and  $E^B$ . Tables 6, 7, and 8 present the results. As in Simulation 3, the portfolio and consumption effects generate contagion. However, assets of different quality get hit in the same way, creating a homogeneous fall in prices. Therefore, Simulation 4 shows the need of something more than agent heterogeneity and market incompleteness to solve the second problem of differential contagion. The collateral-leverage model developed in Section IV will provide a framework to attack both problems of contagion and differential contagion. We shall find liquidity wedge cycles and leverage cycles at work at the same time.

#### IV. Model I: Collateral General Equilibrium

Thus far we have not allowed agents to borrow; they were very limited in how much they could spend on buying what they thought were underpriced assets. Letting the agents use assets as col-

<sup>16</sup> See IMF, GFSR (September 2003).

<sup>17</sup> An alternative mechanism for increasing the liquidity wedge, not considered in this paper but analyzed in Fostel and Geanakoplos (2008), is the appearance of a new investment opportunity available to the optimists but not to the pessimists.

TABLE 6—SIMULATION 4, INCOMPLETE MARKETS WITH 3 ASSETS: PRICES

|          | 1      | $U$    | $D$    | $(U-D)/U$<br>% | $(1-D)/1$<br>% |
|----------|--------|--------|--------|----------------|----------------|
| $\omega$ | 0.0594 | 0.09   | 0.2309 |                |                |
| Asset    |        |        |        |                |                |
| $G$      | 0.7817 | 0.8378 | 0.7431 | 11.3           | 4.9            |
| $B$      | 0.7679 | 0.8230 | 0.7301 | 11.3           | 4.9            |
| $H$      | 0.8477 | 0.9162 | 0.7485 | 18.9           | 12.3           |

TABLE 7—SIMULATION 4, INCOMPLETE MARKETS WITH 3 ASSETS: PORTFOLIO

| Asset | 1      |        | $U$    |        | $D$ |     |
|-------|--------|--------|--------|--------|-----|-----|
|       | $O$    | $P$    | $O$    | $P$    | $O$ | $P$ |
| $G$   | 0.4669 | 0.5331 | 1      | 0      | 1   | 0   |
| $B$   | 0.4675 | 0.5325 | 1      | 0      | 1   | 0   |
| $H$   | 2      | 0      | 0.5219 | 1.4781 | 2   | 0   |

TABLE 8—SIMULATION 4, INCOMPLETE MARKETS WITH 3 ASSETS: CONSUMPTION AND ADJUSTED PROBABILITIES

| Cons.   | 1      | $U$    | $D$    | $UU$   | $UD$   | $DUU$ | $DDU$ | $DUD$ | $DDD$ |
|---------|--------|--------|--------|--------|--------|-------|-------|-------|-------|
| $\mu^O$ |        | 0.894  | 0.106  | 0.893  | 0.108  | 0.798 | 0.096 | 0.095 | 0.011 |
| $x^O$   | 19.2   | 20.5   | 19.2   | 22.5   | 20.7   | 24    | 22.25 | 22.4  | 20.65 |
| $x^P$   | 2000.8 | 1999.5 | 2000.8 | 2001.5 | 2001.5 | 2000  | 2000  | 2000  | 2000  |

lateral to borrow money enables them to take more extreme positions, which will have important consequences for asset pricing. The model we present now includes two critical features. First, agents can use collateral to back promises but are never required to deliver more than the value of their collateral and, second, collateral levels needed to back a given promise are endogenously determined in equilibrium.

### A. The Model

*Time and Uncertainty.*—The model is a finite-horizon general equilibrium model, with time  $t = 1, \dots, T$ . Uncertainty is represented by a tree of date-events or states  $s \in S$ , including a root  $s = 1$ . Each state  $s \neq 1$  has an immediate predecessor  $s^*$ , and each nonterminal node  $s \in S \setminus S_T$  has a set  $S(s)$  of immediate successors. Each successor  $\tau \in S(s)$  is reached from  $s$  via a branch  $\sigma \in B(s)$ ; we write  $\tau = s\sigma$ . We denote the time of  $s$  by the number of nodes  $t(s)$  on the path from 1 to  $s$ . For instance, in our example in Figure 4 we have that the immediate predecessor of  $UU$  is  $UU^* = U$ . The set of immediate successors of  $U$  is  $S(U) = \{UU, UD\}$ . Each of these successors is reached from  $U$  via a branch in the set  $B(U) = \{U, D\}$ . Finally, the time of  $U$  is  $t(U) = 2$ .

*Assets and Collateral.*—A financial contract  $k$  consists of both a promise and collateral backing it, so it is described by a pair  $(A_k, C_k)$ . Collateral consists of durable goods, which will be called assets. The lender has the right to seize as much of the collateral as will make him whole once the loan comes due, but no more.

This paper will focus on a special type of contract. In each state  $s$  its promise is given by  $\phi_s \tilde{\mathbf{1}}_s$ , where  $\tilde{\mathbf{1}}_s \in \mathbb{R}^{S(s)}$  stands for the vector of ones with dimension equal to the number of successors of  $s$ . The contract  $(\phi_s \tilde{\mathbf{1}}_s, C)$  promises  $\phi_s$  units of consumption good in each successor state and is backed by collateral  $C$ . If the collateral is big enough to avoid default, the price of this special

contract is given by  $\phi_s/(1 + r_s)$ , where  $r_s$  is the riskless interest rate. Now, let us be more precise about how the collateral levels are determined.

There is a single consumption good  $x \in \mathbb{R}_+$ .<sup>18</sup> Each asset  $j \in J$  delivers a dividend of the good  $D_{sj}$  in each state  $s \in S$ . The set of assets  $J$  is divided into those assets  $j \in J^c$  that can be used as collateral and those assets  $j \in J \setminus J^c$  that cannot. We shall assume that households are allowed to issue at each state only a noncontingent, one-period promise. Furthermore, we assume that every promise must be backed by collateral so large that payment is guaranteed, ruling out the possibility of default in equilibrium.<sup>19</sup> Thus, holding one unit of collateralizable asset  $j \in J^c$  in state  $s$  permits an agent to issue  $\phi_s$  promises to deliver one unit of the consumption good in each immediate successor state  $t \in S(s)$ , such that

$$(1) \quad \phi_s \leq \min_{t \in S(s)} [p_{tj} + D_{tj}].$$

The *collateral capacity* of one unit of asset  $j$  at state  $s$  is defined by its minimum yield (its price plus the deliveries) in the immediate future states. Notice that the collateral capacity  $\phi_s$  of an asset  $j$  at  $s$  is endogenous, depending on the equilibrium prices  $p_{tj}$ ,  $t \in S(s)$ . In contrast to much of the literature on collateral, our collateral capacity is not taken to be proportional to the current price  $p_{sj}$ . For example, given two branches and a constant interest rate  $r_s$ , if the volatility of future prices  $p_{tj}$  increases while the current price  $p_{sj}$  stays the same, the collateral capacity will fall. The *borrowing capacity* of asset  $j$  at  $s$  is defined by  $\phi_s/(1 + r_s)$ . It depends on the interest rate  $r_s$ , as well as the endogenous collateral capacity of asset  $j$ .

Now we are in position to define one of the key concepts in the paper. Buying one unit of  $j$  on *margin* at state  $s$  means: selling a promise of  $\min_{t \in S(s)} [p_{tj} + D_{tj}]$  using that unit of  $j$  as collateral, and paying  $(p_{sj} - 1/(1 + r_s) \min_{t \in S(s)} [p_{tj} + D_{tj}])$  in cash. The margin of  $j$  at  $s$  is

$$(2) \quad m_{sj} = \frac{p_{sj} - 1/(1 + r_s) \min_{t \in S(s)} [p_{tj} + D_{tj}]}{p_{sj}}.$$

The margin is given by the current asset price net of the amount borrowed using the asset as collateral, as a proportion of the price, i.e., the cash requirement needed to buy the asset today as a proportion of its price. We will denote as *leverage* the inverse of the margin. Similarly, the *loan to value* (LTV) of asset  $j$  is defined as  $1 - m_{sj}$ . The maximal leverage is not only endogenous, but also a forward looking variable; it depends on the current price, on how the asset is going to be priced in the future, and on the interest rate. These endogenous ratios will be of great importance; in particular, they will have a big effect on asset pricing as discussed below.

*Investors.*—Each agent  $i \in I$  is characterized by a utility,  $u^i$ , a discounting factor,  $\delta^i$ , and subjective probabilities,  $q^i$ . We assume that the Bernoulli utility function for consumption in each state  $s \in S$ ,  $u^i: \mathbb{R}_+ \rightarrow \mathbb{R}$  is differentiable, concave, and monotonic. Agent  $i$  assigns subjective probability  $q_s^i$  to the transition from  $s^*$  to  $s$ ; naturally,  $q_1 = 1$ . Letting  $\bar{q}_s^i$  be the product of all  $q_s^i$  along the path from 1 to  $s$ , the von Neumann–Morgenstern expected utility to agent  $i$  is

$$(3) \quad U^i = \sum_{s \in S} \bar{q}_s^i (\delta^i)^{t(s)-1} u^i(x_s).$$

<sup>18</sup> Considering a single consumption good greatly simplifies notation without loss of generality, since the focus here will be primarily on asset prices.

<sup>19</sup> Geanakoplos (2003) showed that with heterogeneous priors and two successors states, even if agents were allowed to use asset  $j$  to collateralize any promise of the form  $\phi \mathbf{1}$ , they would never choose  $\phi$  so big as to permit any default. We shall see that even in the absence of default, there are inefficiencies in international financial markets.

Each investor  $i$  begins with an endowment of the consumption good  $e_s^i \in \mathbb{R}_+$  in each state  $s \in S$ , and an endowment of assets at the beginning  $y_{1^*}^i \in \mathbb{R}_+^J$ . We assume that all assets and the consumption good are present,  $\sum_{i \in I} y_{1^*}^i \gg 0$  and  $\sum_{i \in I} e_s^i > 0$ ,  $\forall s \in S$ . Given asset prices and interest rates  $((p_s, r_s), s \in S)^{20}$ , each agent  $i \in I$  chooses consumption,  $x_s$ , asset holdings,  $y_{sj}$ , and borrowing (lending),  $\phi_s$ , in order to maximize utility (3) subject to the budget set defined by

$$B^i(p, r) = \{(x, y, \phi) \in \mathbb{R}_+^S \times \mathbb{R}_+^{SJ} \times \mathbb{R}^S : \forall s,$$

$$(x_s - e_s^i) + \sum_{j \in J} p_{sj}(y_{sj} - y_{s^*j}) \leq \frac{1}{1 + r_s} \phi_s - \phi_{s^*} + \sum_{j \in J} y_{s^*j} D_{sj},$$

$$\phi_s \leq \sum_{j \in J^c} y_{sj} \min_{t \in S(s)} [p_{tj} + D_{tj}]\}.$$

In each state  $s$ , expenditures on consumption minus endowments of the good, plus total expenditures on assets minus asset holdings carried over from the last period, can be at most equal to the money borrowed selling promises, minus the payments due at  $s$  from promises made in the previous period, plus the total asset deliveries. Notice that there is no sign constraint on  $\phi_s$ ; a positive (negative)  $\phi_s$  indicates the agent is selling (buying) promises or, in other words, borrowing (lending) money. The last line displays the *collateral constraint*: the total amount of promises made at  $s$  cannot exceed the total collateral capacity of all collateralizable asset holdings.

*Collateral Equilibrium.*—A *collateral equilibrium* in this economy is a set of prices and holdings such that

$$((p, r), (x^i, y^i, \phi^i)_{i \in I}) \in \mathbb{R}_+^{SJ} \times \mathbb{R}_+^S \times (\mathbb{R}_+^S \times \mathbb{R}_+^{SJ} \times \mathbb{R}^S)^I : \forall s,$$

$$\sum_{i \in I} (x_s^i - e_s^i) = \sum_{i \in I} \sum_{j \in J} y_{s^*j}^i D_{sj},$$

$$\sum_{i \in I} (y_{sj}^i - y_{s^*j}^i) = 0, \forall j,$$

$$\sum_{i \in I} \phi_s^i = 0,$$

$$(x^i, y^i, \phi^i) \in B^i(p, r), \forall i,$$

$$(x, y, \phi) \in B^i(p, r) \Rightarrow U^i(x) \leq U^i(x^i), \forall i.$$

Markets for the consumption good, assets, and promises clear in equilibrium, and agents optimize their utility constrained to their budget sets as defined above. A collateral equilibrium always exists under all the described assumptions in this model, as shown by Geanakoplos and Zame (1998). As is well known, this is not true for the standard general equilibrium model with incomplete markets, since equilibrium may fail to exist without a bound on promises; the best

<sup>20</sup> The consumption good is the numeraire, so  $p_{sx} = 1$ .

result in the standard model is only generic existence. Collateral requirements fix this problem since they place (an endogenous) bound on promises.

### B. Asset Pricing

An asset's price reflects its future returns, but also its ability to be used as collateral to borrow money. Consider a collateral equilibrium in which an agent  $i$  holds an asset  $j$  at state  $s \in S$ ,  $y_{sj}^i > 0$ , and suppose  $i$  consumes a positive amount in each state. Suppose, first, that asset  $j$  cannot be used as collateral. Then, the price of asset  $j$  equals its *Payoff Value to  $i$* ,  $PV_{sj}^i$ ,

$$(4) \quad p_{sj} = PV_{sj}^i \equiv \frac{\sum_{\sigma \in B(s)} \delta^i q_{s\sigma}^i [p_{s\sigma j} + D_{s\sigma j}] du^i(x_{s\sigma}^i)/dx}{du^i(x_s^i)/dx},$$

i.e., the normalized expected marginal utility of its future payoff to agent  $i$  from state  $s$ . This equation remains true if  $j$  can be used as collateral but the collateral constraint for  $i$  is not binding at  $s$ . But if the collateral constraint is binding, the equation need not hold. Typically, people who start to buy an asset on margin do so because the payoff value to them is much higher than the price and they would like to get their hands on as much of the asset as possible. But as they buy more, the marginal utility falls, and because they can leverage their purchases, they continue to buy even past the point where the payoff value falls below the price, leading to the counterintuitive conclusion that

$$(5) \quad p_{sj} > PV_{sj}^i = \frac{\sum_{\sigma \in B(s)} \delta^i q_{s\sigma}^i [p_{s\sigma j} + D_{s\sigma j}] du^i(x_{s\sigma}^i)/dx}{du^i(x_s^i)/dx},$$

as we shall prove in the pricing lemmas below.

Similarly, if the collateral borrowing constraint for  $i$  is not binding at  $s$ , then the first-order condition for borrowing holds:

$$(6) \quad \frac{1}{1 + r_s} = \frac{\sum_{\sigma \in B(s)} \delta^i q_{s\sigma}^i du^i(x_{s\sigma}^i)/dx}{du^i(x_s^i)/dx}.$$

However, when an asset can be used as collateral, and the collateral constraint is binding, the situation is quite different. Agent  $i$  cannot take out an additional loan unless he holds additional collateral. Thus, even if the marginal disutility of repaying the loan is less than the marginal utility of the money borrowed, it may just be impossible to borrow more money:

$$(7) \quad \frac{1}{1 + r_s} > \frac{\sum_{\sigma \in B(s)} \delta^i q_{s\sigma}^i du^i(x_{s\sigma}^i)/dx}{du^i(x_s^i)/dx}.$$

**DEFINITION:** Define the liquidity wedge  $\omega_s^i$  for agent  $i$  at  $s$  by

$$(8) \quad \frac{1}{1 + \omega_s^i} \frac{1}{1 + r_s} = \frac{\sum_{\sigma \in B(s)} \delta^i q_{s\sigma}^i du^i(x_{s\sigma}^i)/dx}{du^i(x_s^i)/dx}.$$

The liquidity wedge is the excess interest agent  $i$  would be willing to promise in state  $s$  to get a loan if he did not have to put up any collateral, but was indeed committed to fully paying. Since

there are agents willing to lend at the equilibrium interest rate  $r_s$ , the liquidity wedge measures the gap between the rates borrowers are willing to pay and lenders are willing to take if payment is guaranteed.

The right-hand side of equation (6) defines the interest rate at which each agent  $i$  is willing to borrow or lend in state  $s$ . If there were a fully functioning loan market with no repayment problems, these numbers would be the same for all agents, namely, equal to the market interest rate, as in equation (6). But with the loan market constrained by the collateral requirement, or without any loan market at all, as in Simulations 3 and 4, these numbers could be wedged apart. The liquidity wedge for the economy in state  $s$  is the maximum ratio of the right-hand side of equation (6), taken over all pairs of agents. The liquidity wedge is thus a measure of the inefficiency in the credit market resulting from the difficulty of enforcing payments and the scarcity of collateral.

**DEFINITION:** Define the effective collateral capacity  $\phi_{sj}^i$  as the debt of agent  $i$  backed by a marginal unit of asset  $j$

$$\phi_{sj}^i = \begin{cases} 0 & \text{if } j \notin J^c \text{ or if the collateral constraint is not binding at } s \text{ for } i, \\ \min_{t \in S(s)} [p_{tj} + D_{tj}] & \text{otherwise.} \end{cases}$$

**DEFINITION:** The collateral value of asset  $j$  in state  $s$  to agent  $i$  is the marginal benefit from being able to take out loans backed by asset  $j$ :

$$(9) \quad CV_{sj}^i \equiv \left[ \frac{1}{1+r_s} - \frac{1}{1+\omega_s^i} \frac{1}{1+r_s} \right] \phi_{sj}^i = \frac{1}{1+r_s} \frac{\omega_s^i}{1+\omega_s^i} \phi_{sj}^i.$$

The collateral value reflects the asset's marginal contribution to agent  $i$ 's liquidity. This contribution depends first on the asset's effective collateral capacity  $\phi_{sj}^i$ , second on how valuable liquidity is to agent  $i$  as measured by the liquidity wedge  $\omega_s^i$ , and third on the interest rate  $r_s$ . Note that the collateral value of an asset rises as the liquidity wedge  $\omega_s^i$  rises. We are ready to explain inequality (5).

**PRICING LEMMA 1:** Suppose that  $y_{sj}^i > 0$  for the marginal buyer  $i$ . Then,

$$p_{sj} = PV_{sj}^i + CV_{sj}^i.$$

The price equals the sum of the payoff value and collateral value.<sup>21</sup>

<sup>21</sup> These concepts relate to the standard concept of fundamental value of an asset in the following way. Define the fundamental value of an asset  $j$  at  $s$  as

$$FV_{sj}^i = \frac{\sum_{\gamma \in \Gamma(s)} (\delta^i)^{t(\gamma)-t(s)} \bar{q}_\gamma^i D_\gamma^j du^i(x_\gamma^i)/dx}{du^i(x_s^i)/dx},$$

where  $\Gamma(s)$  is the set of all the successors (not only immediate), and  $\bar{q}_\gamma^i$  is the product of all  $q_{st}^i$  along the path from  $s$  to  $\gamma$ . If the asset cannot be used as collateral, then  $p_{sj} = PV_{sj}^i = FV_{sj}^i$ . However, if the asset can be used as collateral, then typically  $p_{sj} > PV_{sj}^i > FV_{sj}^i$ .



PROOF:

The first-order condition that must hold in equilibrium if  $i$  holds asset  $j$  is that the marginal utility of the cash payment necessary to buy  $j$  is equal to the expected marginal utility of the unencumbered payoff, i.e., the return on  $j$  less the repayment of the debt:

$$(10) \quad p_{sj} - \frac{1}{1+r_s} \phi_{sj}^i = \frac{\sum_{\sigma \in B(s)} \delta_i q_{s\sigma}^i [p_{s\sigma j} + D_{s\sigma j} - \phi_{sj}^i] du^i(x_{s\sigma}^i)/dx}{du^i(x_s^i)/dx}.$$

The pricing lemma follows from the definitions of  $PV_{sj}^i$  and  $CV_{sj}^i$  and equations (8) and (10):

$$\begin{aligned} p_{sj} &= PV_{sj}^i + \frac{1}{1+r_s} \phi_{sj}^i - \frac{1}{1+\omega_s^i} \frac{1}{1+r_s} \phi_{sj}^i \\ &= PV_{sj}^i + CV_{sj}^i. \end{aligned}$$

Another convenient way to state Pricing Lemma 1 is as follows. Let  $\mu_s^i \in \mathbb{R}^{S(s)}$  be the risk adjusted probabilities agent  $i$  attaches to each branch out of  $s$ ,

$$(11) \quad \mu_{s\sigma}^i = \frac{du^i(x_{s\sigma}^i)/dx}{\sum_{\tau \in B(s)} q_{s\tau}^i du^i(x_{\tau\sigma}^i)/dx} q_{s\sigma}^i, \sigma \in B(s).$$

If consumption is not too different across states, then the  $\mu_{s\sigma}^i$  will be very close to the subjective probabilities  $q_{s\sigma}^i$ . Let  $\tilde{\mathbf{1}}_s \in \mathbb{R}^{S(s)}$  be the vector of ones with dimension equal to the number of successors of  $\sigma$ . Let  $\mathbf{A}_{sj} \in \mathbb{R}^{S(s)}$  be the vector of payoffs of asset  $j$  in each state following  $s$ ,  $\mathbf{A}_{sj} = [p_{s\sigma j} + D_{s\sigma j}]$ . Then, the payoff value to  $i$  of  $j$  at  $s$  is given by

$$(12) \quad PV_{sj}^i = \frac{1}{1+r_s} \frac{1}{1+\omega_s^i} \mu_s^i \cdot \mathbf{A}_{sj}$$

and the collateral value is given by

$$(13) \quad CV_{sj}^i = \frac{1}{1+r_s} \frac{\omega_s^i}{1+\omega_s^i} \mu_s^i \cdot \tilde{\mathbf{1}}_s \phi_{sj}^i.$$

Combining equations (12), (13), and the identity  $\mu_s^i \cdot \tilde{\mathbf{1}}_s = 1$  with Pricing Lemma 1 gives:

PRICING LEMMA 2:

$$\begin{aligned} p_{sj} &= \frac{1}{1+r_s} \frac{1}{1+\omega_s^i} \mu_s^i \cdot \mathbf{A}_{sj} + \frac{1}{1+r_s} \frac{\omega_s^i}{1+\omega_s^i} \phi_{sj}^i \\ &= \frac{1}{1+r_s} \frac{1}{1+\omega_s^i} [\mu_s^i \cdot (\mathbf{A}_{sj} - \phi_{sj}^i \tilde{\mathbf{1}}_s)] + \frac{1}{1+r_s} \phi_{sj}^i. \end{aligned}$$

From Pricing Lemma 1 we see that two assets with the same payoff values may sell for very different prices if their collateral values differ.

From the first line of Pricing Lemma 2 we see that, all other things being equal, an increase in the liquidity wedge decreases the payoff value and increases the collateral value. Thus, for two

assets that continue to have (nearly) equal payoff values, an increase in the liquidity wedge will increase the gap in price between the asset with high collateral capacity and the asset with low collateral capacity.

From the second line of Pricing Lemma 2, we see that so long as the interest rate  $r_s$ , the risk adjusted probabilities  $\mu_s^i$ , and the effective collateral capacities  $\phi_{sj}^i$  remain approximately unchanged, an increase in the liquidity wedge  $\omega_s^i$  lowers asset prices, since  $(A_{sj} - \phi_{sj}^i \tilde{\mathbf{1}}_s) \geq \mathbf{0}$ .

In all our simulations, the  $\mu_s^i$  are very close to the subjective probabilities  $q_s^i$ .<sup>22</sup> The reason is that we are considering the anxious economy, where agents' consumption is not drastically altered, and certainly never driven anywhere near zero.

In Simulations 1 to 4, collateral is not allowed,  $\phi_{sj}^i = 0$ , and so by the first line of Pricing Lemma 2, asset prices closely track their expected payoffs, discounted by the interest rate and the liquidity wedge. The expected payoffs of the  $E$  assets are the same from 1,  $U$ , and  $D$ , and hence we can expect their prices to depend on the interest rate and on the liquidity wedge.

In all our simulations, the interest rate does not vary much between states. Indeed, the interest rate does not play a significant role in the anxious economy. Hence, the variation in the prices of  $E$  is almost entirely explained by the liquidity wedge. In Simulations 1 and 2, the liquidity wedge is zero, and there is virtually no variation in the price of  $E$  between  $U$  and  $D$ , and no contagion. In Simulations 3 and 4, the liquidity wedge jumps (from 1 to  $D$ ) from 0.07 to 0.24 and 0.06 to 0.23, respectively, and the price of  $E$  drops at  $D$ , and so there is contagion.

### C. The Leverage Cycle and Contagion

In this section we will extend our example in order to understand the role of collateral in contagion. Simulation 5 solves the equilibrium for the same assets and investor characteristics as Simulation 3, except that now  $E$  can be used as collateral to borrow money, and hence can be leveraged. For simplicity, we will assume that  $H$  cannot be used as collateral. Table 9 presents the equilibrium prices. As before, there is contagion due to the portfolio effect on the liquidity wedge cycle. Is there something different this time?

The conventional wisdom is that leverage causes agents to lose more money during crises, making asset prices even lower. On the contrary, we find that during the anxious economy, at  $D$ , asset prices are higher than they would have been without collateral. Yet leverage still causes bigger price crashes from 1 to  $D$  (but not from  $U$  to  $D$ ).

In general, since collateral facilitates borrowing and leverage, it tends to reduce the liquidity wedge. It also creates a new source of value, the collateral value. This is why in every node we observe higher asset prices in Simulation 5 than in Simulation 3. Across nodes, however, leverage is not the same. In normal times leverage endogenously becomes high (because next period's price volatility is low), raising asset prices even further. In anxious times leverage endogenously becomes low (because next period's price volatility is high), causing asset prices to fall. We call this the leverage cycle. The underlying causes of the liquidity wedge cycle—fluctuating uncertainty and disagreement—are also causes of the leverage cycle; that is why they run in parallel. The leverage cycle thus reinforces the liquidity wedge cycle.

Pricing Lemmas 1 and 2 will explain this. Table 10 provides disaggregated information about price components, the liquidity wedge, and margin requirements in equilibrium at each node. First, notice that the risk-adjusted probabilities of the optimist in both simulations are very close to his subjective probabilities, because consumption does not vary much. Second, the interest rate remains nearly constant (close to zero) across every state. Hence, by Pricing Lemma 2, asset

<sup>22</sup> The one exception is for the optimist in Simulation 2. But the pessimist is also a marginal buyer of the asset in that simulation, and his  $\mu_s^i$  are indeed very close to his  $q_s^i$ .

TABLE 9—SIMULATION 5, INCOMPLETE MARKETS WITH COLLATERAL: PRICES AND INTEREST RATE

| Asset | 1      | $U$     | $D$    | $(U - D)/U$<br>% | $(1 - D)/1$<br>% |
|-------|--------|---------|--------|------------------|------------------|
| $E$   | 0.8511 | 0.8695  | 0.7416 | 14.7             | 12.9             |
| $H$   | 0.9316 | 0.9985  | 0.7306 | 26.8             | 21.6             |
| $r$   | 0.0000 | -0.0015 | 0.0005 |                  |                  |

prices are explained entirely by expected payoffs, effective collateral capacities, and the liquidity wedge.

At  $U$  the price of  $E$  in Simulation 5, with collateral, and in Simulation 3, without collateral, is high and almost the same. First, the payoff value is high and essentially the same because the liquidity wedge,  $\omega_U$ , is low and nearly the same in both simulations. Second, the collateral value in Simulation 3 is zero by definition and in Simulation 5 it is small (since after good news at  $U$ , the liquidity wedge is small, and the collateral capacity is low in the second period).

At  $D$  the price of  $E$  is low in both simulations, but not quite as low in Simulation 5. The payoff value is low and nearly the same in both simulations, since the liquidity wedge is high and nearly the same in both simulations.<sup>23</sup> (In Simulation 5 the portfolio and consumption effects and endogenous margins cause the liquidity wedge to spike up from 0.044 at  $U$  and 0.04 at 1 to 0.25 at  $D$ .<sup>24</sup>) However, the collateral value becomes significant in Simulation 5, because the liquidity wedge is high. There is no collateral value in Simulation 3. This collateral value explains why the price at  $D$  is bigger when there is collateral, and hence explains why the gap between  $U$  and  $D$  is smaller with collateral than without.<sup>25</sup>

At 1 the price is higher with collateral than without for three reasons. First, the payoff value is higher than it was without collateral since the liquidity wedge is lower in the good phase of the leverage cycle. Second, the payoff value is also higher due to the presence of future collateral values, which raises future prices. Third, the collateral value is high, even though the liquidity wedge is only moderate, because the collateral capacity is high in the good phase of the leverage cycle (since the asset values at  $U$  and  $D$  are still high). The leverage of  $E$  at 1 is  $1/0.1286 = 7.8$ , while at  $D$  it is only  $1/0.8651 = 1.2$ . Looking in hindsight from  $D$  at the very high asset prices in node 1 attributable to leverage might well lead the press to talk of asset price bubbles.

To sum up, leverage is not necessary for contagion to occur in equilibrium, as shown by Simulation 3. Portfolio and consumption effects are sufficient to generate a liquidity wedge cycle that affects payoff values, and hence prices. During normal times, the liquidity wedge is small and hence payoff values are large, while during anxious times the liquidity wedge expands, lowering payoff values and hence prices.

<sup>23</sup> The “wealth effect” implicit in other models that focus on the crisis stage has almost no bite in the anxious stage at  $D$ : it is true that leverage at 1 has a negative consumption effect at  $D$ , since it causes optimists to lose more money. But this is almost exactly offset by a positive consumption effect due to the possibility of borrowing again. The fall in consumption from  $U$  to  $D$  of 9 percent that we saw in Simulation 3 is barely worsened to 10 percent by leverage in Simulation 5. Hence, the liquidity wedge in the two simulations is nearly the same, and the payoff value at  $D$  is only slightly lower with collateral than without.

<sup>24</sup> The increased uncertainty arising at  $D$  about the payoff of  $H$  would naturally lower the collateral capacity (raise the margin) of  $H$ , thus decreasing leverage, were it a collateralizable asset. That is precisely the effect studied in Geanakoplos (2003). This in turn would exacerbate the increase in the liquidity wedge caused by the portfolio effect at  $D$ . In our model in Simulation 5 the margin of  $E$  also increases from 1 to  $D$  (though not from  $U$  to  $D$ ). From Table 10 we see that  $m_{DE} = 0.86 > m_{1E} = 0.12$ . This is a consequence of assuming a three-period model with exclusively terminal payoffs; as time approaches the end, uncertainty must increase in the absence of news. (Thus,  $m_{UE} = 0.88 > m_{1E} = 0.12$  as well.) Having obtained a leverage cycle through asset  $E$ , we suppressed the leverage cycle via asset  $H$  by assuming for simplicity that  $H$  cannot be used as collateral.

<sup>25</sup> One may wonder if leverage could destroy contagion at  $D$  since the collateral value might rise enough to offset the fall in the payoff values. But this possibility is ruled out by the second part of Pricing Lemma 2.

TABLE 10—SIMULATION 5, INCOMPLETE MARKETS WITH COLLATERAL: PRICE COMPONENTS, LIQUIDITY PREFERENCE, AND MARGINS

|          |      | 1      | $U$    | $D$    | $(U - D)/p_U$<br>% | $(1 - D)/p_1$<br>% |
|----------|------|--------|--------|--------|--------------------|--------------------|
| $\omega$ |      | 0.0403 | 0.0446 | 0.2515 |                    |                    |
| Assets   |      |        |        |        |                    |                    |
| $E$      | $PV$ | 0.8223 | 0.8655 | 0.7215 | 16.6               | 11.8               |
|          | $CV$ | 0.0287 | 0.0043 | 0.0201 | -1.8               | 1.1                |
|          | $m$  | 0.1286 | 0.8852 | 0.8651 |                    |                    |
| $H$      | $PV$ | 0.9316 | 0.9985 | 0.7306 | 26.8               | 21.6               |
|          | $CV$ | 0.0000 | 0.0000 | 0.0000 |                    |                    |
|          | $m$  | 1.0000 | 1.0000 | 1.0000 |                    |                    |

So what does collateral, and the possibility of leverage, add to this scenario? It generates a bigger price crash, not due to *asset undervaluation* during anxious times but due to *asset overvaluation* during normal times. There are two main reasons for this. First, the leverage cycle amplifies the liquidity wedge cycle caused by the portfolio effect. Second, when we add collateral explicitly into the model, we add a new channel through which liquidity affects asset prices: the collateral value. As we saw, however, the amplification is done through the increase in the price at 1, not through a decrease in the price at  $D$ .<sup>26</sup>

Finally, both simulations provide a solution to the first problem and, in particular, rationalize Stylized Fact 1. Even without problems in emerging market fundamentals, a bad shock to the high yield sector could have negative spillovers on emerging markets.

#### D. Robustness

The fundamental source of contagion is the portfolio effect, namely, bad news about  $H$  gives optimists an opportunity to hold it at attractive levels, reducing the money they can put into  $E$ . In Simulation 5, optimists hold no  $H$  after good news at  $U$ , and all of  $H$  after bad news at  $D$ . Table 11 provides portfolio holdings and consumption at each node.

This corner solution gives an extreme form of the portfolio effect. One may wonder how robust contagion is to other regimes, where, for example, pessimists and optimists may both be marginal buyers of all the assets. Different parameter values will change asset holdings in equilibrium, allowing us to explore this question. It turns out that the simulation is not just a fluke. In fact, contagion is quite robust to other parameter choices. Two crucial parameters are investors' beliefs and wealth. So, let us keep the rest of the values at the original levels and fix  $q^O = 0.9$  and  $e_s^O = 20$ . Define  $q^O - q^P$  as the disagreement and  $e^P - e^O$  as wealth gap between investors. Figure 5 presents a grid of simulations. In all the regions numbered from 1 to 11, contagion holds in equilibrium. The different regions correspond to different regimes in terms of asset holdings and whether collateral constraints are binding or not. Region 1 corresponds to Simulation 5. But contagion holds also in less extreme portfolio regimes. For example, in regime 8 optimists and pessimists both hold  $H$  in both states in the second period, but optimists still hold more  $H$  at  $D$  than at  $U$ , so the portfolio effect is still present. The only regions in which contagion

<sup>26</sup> In the conventional story,  $H$  is leveraged and the bad news about  $H$  induces investors who are leveraged in  $H$  to sell  $E$ , causing its price to fall more than if there had been no leverage. However, when Simulation 5 was extended to allow  $H$ , and not only  $E$ , to be used as collateral, all the results in this section remained intact. In particular, the price of  $E$  is *higher* at  $D$  when both assets can be used as collateral than when not. As we have said, state  $D$  corresponds to an anxious economy, not a crisis economy.

TABLE 11—SIMULATION 5, INCOMPLETE MARKETS WITH COLLATERAL: ALLOCATIONS

|                         | 1                 | <i>U</i>         | <i>D</i>         | <i>UU</i> | <i>UD</i> | <i>DUU</i> | <i>DDU</i> | <i>DUD</i> | <i>DDD</i> |
|-------------------------|-------------------|------------------|------------------|-----------|-----------|------------|------------|------------|------------|
| Cons.                   |                   |                  |                  |           |           |            |            |            |            |
| $x^O$                   | 19.7              | 20.7             | 18.7             | 21.8      | 20        | 23.8       | 22         | 22.2       | 20.4       |
| $x^P$                   | 2000.3            | 1999.3           | 2001.3           | 2002.2    | 2002.2    | 2000.2     | 2000.2     | 2000.2     | 2000.2     |
| Adj. Prob               |                   |                  |                  |           |           |            |            |            |            |
| $\mu^O$                 |                   | 0.891            | 0.110            | 0.892     | 0.108     | 0.798      | 0.096      | 0.095      | 0.012      |
| Portfolio               |                   |                  |                  |           |           |            |            |            |            |
| $y_H^O$                 | 2                 | 0                | 2                |           |           |            |            |            |            |
| $y_E^O$                 | 2                 | 2                | 2                |           |           |            |            |            |            |
| $y_H^P$                 | 0                 | 2                | 0                |           |           |            |            |            |            |
| $y_E^P$                 | 0                 | 0                | 0                |           |           |            |            |            |            |
| Loans                   |                   |                  |                  |           |           |            |            |            |            |
| $\varphi^O(-\varphi^P)$ | $2 \times (0.74)$ | $2 \times (0.1)$ | $2 \times (0.1)$ |           |           |            |            |            |            |

breaks down are the two lower regions 12 and 13, where  $q^P = 0.8999$  and  $q^O - q^P$  is near zero. Of course, at the origin we are back to the case of a representative agent. Table 12 describes all these regimes showing at each node what the asset holdings are for each type of investor and whether the borrowing constraint is binding. A “—” indicates closed credit markets (there is no borrowing or lending). In regimes 1 to 11, if credit markets are active, optimists always borrow and pessimists always lend, and the contrary is true in regimes 12 and 13.

With complete markets, an increase in pessimists’ wealth would destroy contagion. With incomplete markets, contagion holds regardless of the wealth gap (provided there is disagreement between agents). In fact, the degree of contagion increases (given a disagreement level) with the wealth gap, measured both as the gap between *U* and *D* and the fall from 1 to *D*, as shown in Figures 6 and 7.<sup>27</sup> The reason for this is that richer pessimists lend more elastically, making for lower interest rates in equilibrium. With lower interest rates, borrowing capacities go up (even with constant collateral capacities) and hence the optimist’s ability to hold more extreme portfolio positions.

### E. Leverage Cycles and Flight to Collateral

By now it is commonly accepted that during crises “high quality” assets fall less in price than “low quality” assets. We observed this in our emerging market data during closures. The question is why. In Simulation 4, we saw that without collateral the liquidity wedge cycle caused the same decline in both emerging market assets. Although the leverage cycle was not necessary for contagion, it will now play the prominent role in explaining differential contagion.

Simulation 6 solves the equilibrium for the same parameters as in Simulation 4, except that now both emerging market assets can be used as collateral. Without loss of generality we still assume that this is not the case for *H*. Tables 13 and 14 present the results.

The portfolio and consumption effects are still present, and hence so is contagion. However, simulation 6 exhibits a new thing: *differential contagion*. The price of  $E^B$  falls more than the price of  $E^G$  from *U* to *D* and from 1 to *D*.

The key is that different assets experience different leverage cycles, because they all have their own endogenous margins (collateral capacities) in equilibrium. The liquidity wedge has a common effect on all asset values, but the collateral values also depend on the idiosyncratic

<sup>27</sup> We show the degree of contagion for only two disagreement levels; more information is available upon request.

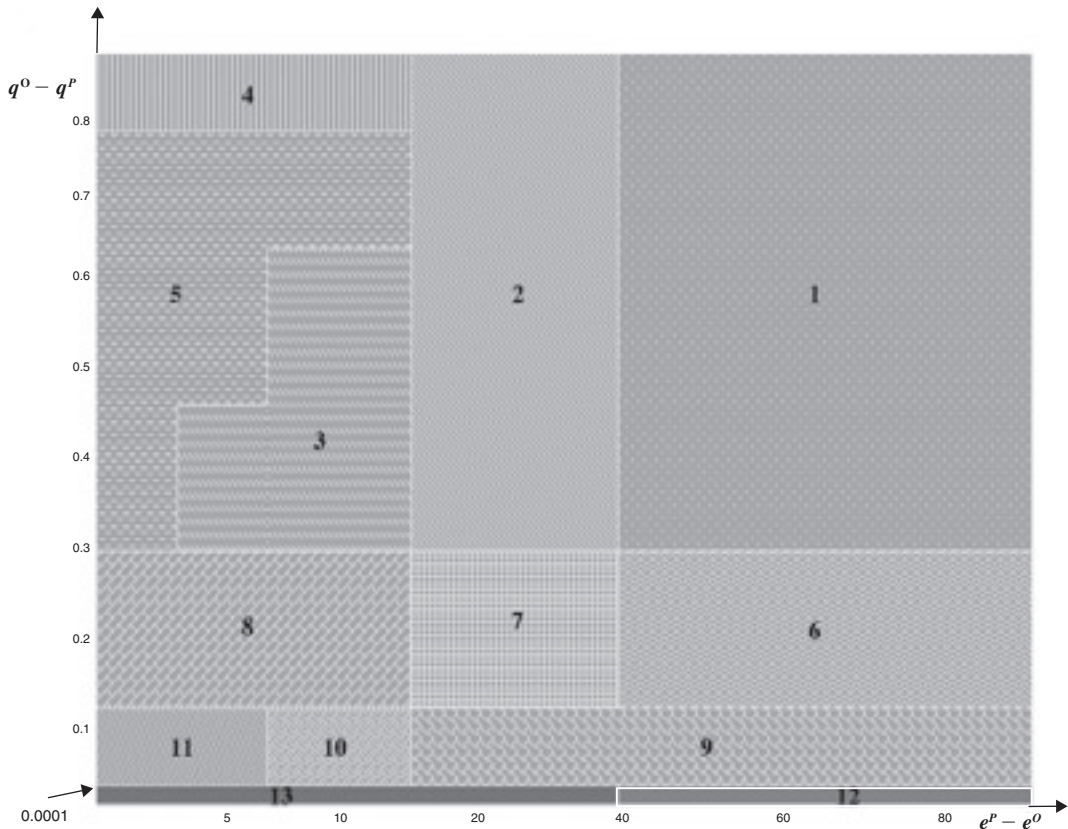


FIGURE 5. ROBUSTNESS ANALYSIS

margins. The differentiated behavior in collateral values explains the differential fall in prices; the changes in payoff values for both assets are virtually the same across all states. From  $U$  to  $D$  the fall in payoff values of 16.5 percent for both assets is cushioned by an *increase* of 3.7 percent in the collateral value for  $E^G$ , but only of 0.9 percent for  $E^B$ . From 1 to  $D$  the difference is even more drastic, since the common fall in payoff values of approximately 12 percent is cushioned by an *increase* in the collateral value of 1 percent for  $E^G$  but exacerbated by a further *decrease* in the collateral value of 2.2 percent for  $E^B$ .<sup>28</sup>

More precisely, at  $D$  the collateral capacities of  $E^G$  and  $E^B$  are quite different, giving rise to LTVs of  $1 - m_{DG} = 0.26$  and  $1 - m_{DB} = 0.07$ , respectively. The high liquidity wedge  $\omega_D = 0.24$  and the different borrowing capacities give rise to different collateral values of  $CV_{DG} = 0.04$  and  $CV_{DB} = 0.01$ . At  $U$  the collateral capacities are also very different, but the liquidity wedge is so low ( $\omega_U = 0.04$ ) that the collateral values are negligible and thus virtually the same,  $CV_{UG} = 0.007$  and  $CV_{UB} = 0.002$ . At 1, the endogenous LTVs are  $1 - m_{1G} = 0.89$  and  $1 - m_{1B} = 0.88$ , which though big are very similar. Combined with a low liquidity wedge ( $\omega_1 = 0.04$ ), they lead to very similar, though not negligible, collateral values of  $CV_{1G} = 0.03$  and  $CV_{1B} = 0.028$ .

<sup>28</sup> As before, it can be shown that the result is robust to different parameter specifications. We will save the reader from this discussion since there is nothing conceptually new from the analysis already presented.



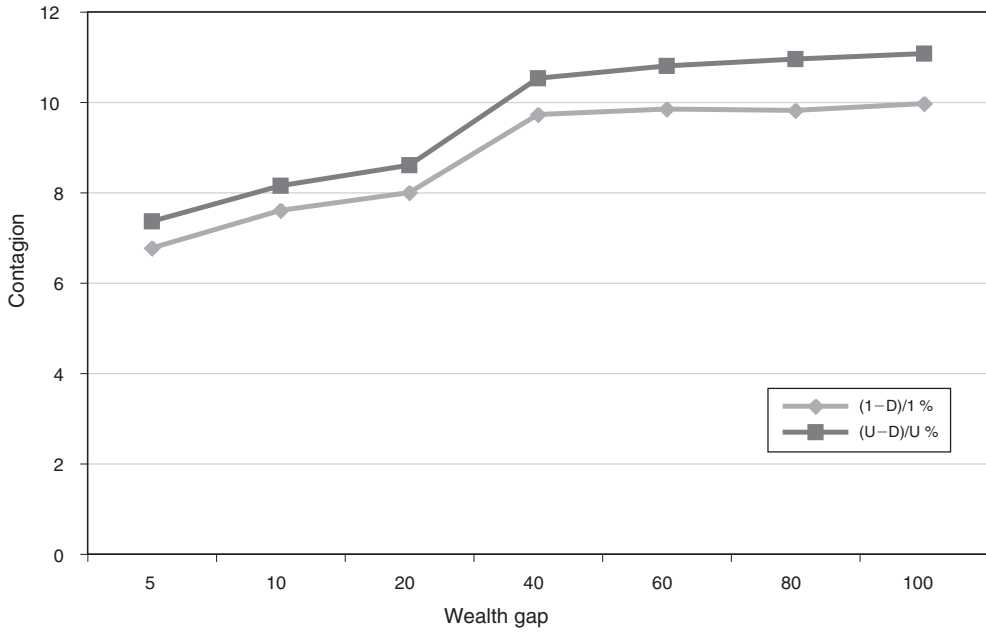


FIGURE 6. CONTAGION FOR DISAGREEMENT LEVEL 0.2

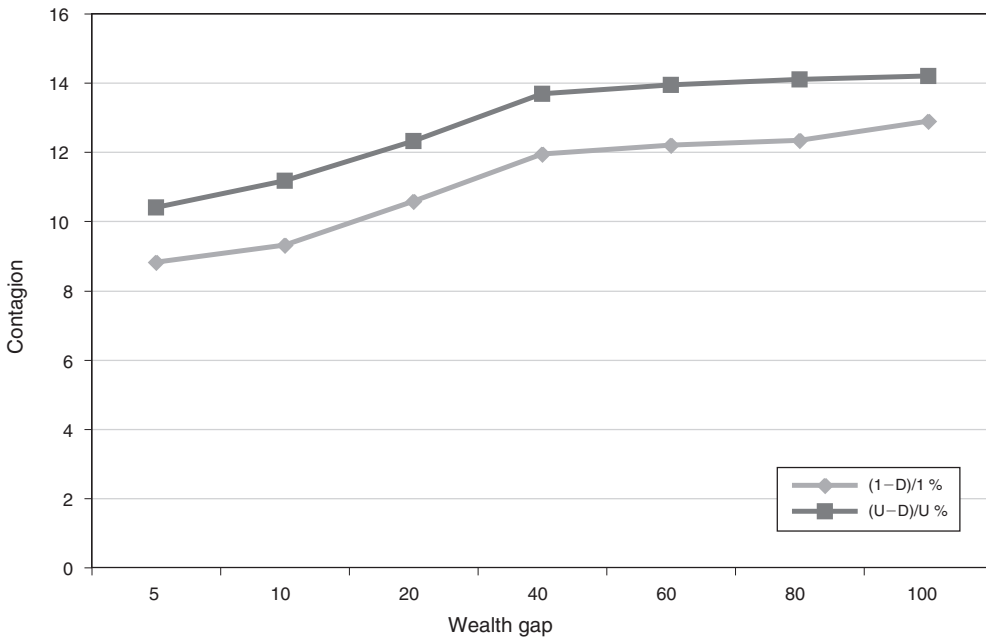


FIGURE 7. CONTAGION FOR DISAGREEMENT LEVEL 0.4

TABLE 12—ROBUSTNESS, DESCRIPTION OF REGIMES

| Regime | Node 1    |            |                      | Node U    |            |                      | Node D    |            |                      |
|--------|-----------|------------|----------------------|-----------|------------|----------------------|-----------|------------|----------------------|
|        | Optimists | Pessimists | Borrowing constraint | Optimists | Pessimists | Borrowing constraint | Optimists | Pessimists | Borrowing constraint |
| 1      | $E, H$    | —          | $B$                  | $E$       | $H$        | $B$                  | $E, H$    | —          | $B$                  |
| 2      | $E, H$    | —          | $B$                  | $E, H$    | $H$        | $B$                  | $E, H$    | —          | $B$                  |
| 3      | $E, H$    | —          | $B$                  | $E$       | $H$        | $NB$                 | $E, H$    | —          | $B$                  |
| 4      | $E, H$    | —          | $NB$                 | $E, H$    | $H$        | $B$                  | $E, H$    | —          | $B$                  |
| 5      | $E, H$    | —          | $NB$                 | $E, H$    | $H$        | —                    | $E, H$    | —          | $B$                  |
| 6      | $E, H$    | —          | $B$                  | $E$       | $H$        | $B$                  | $E, H$    | $H$        | $B$                  |
| 7      | $E, H$    | —          | $B$                  | $E$       | $H$        | —                    | $E, H$    | $H$        | $B$                  |
| 8      | $E, H$    | —          | $B$                  | $E, H$    | $H$        | —                    | $E, H$    | $H$        | $B$                  |
| 9      | $E, H$    | $H$        | $B$                  | $E$       | $H$        | $B$                  | $E, H$    | $H$        | $B$                  |
| 10     | $E, H$    | $H$        | $B$                  | $E$       | $H$        | —                    | $E, H$    | $H$        | $B$                  |
| 11     | $E, H$    | $H$        | $B$                  | $E, H$    | $H$        | —                    | $E, H$    | $H$        | $B$                  |
| 12     | $E, H$    | $E, H$     | —                    | $E, H$    | $E, H$     | —                    | $E, H$    | $E, H$     | —                    |
| 13     | $E, H$    | $E, H$     | —                    | $E, H$    | $E, H$     | —                    | $E, H$    | $E, H$     | $B$                  |

We say that there is *flight to collateral* when there is an increase in the spread between assets due to different collateral values. This happens when: (a) the liquidity wedge is high, and (b) the dispersion of margins between assets is high. During a flight to collateral, investors would rather *buy* those assets that enable them to borrow money more easily (lower margins). The other side of the coin is that investors who need to raise cash get more by *selling* those assets on which they did not borrow money (higher margins) because the sales revenues net of loan repayments are higher.

Traditionally, the price deterioration of low quality assets is explained in terms of “flight to quality” type of arguments: an increase in risk aversion lowers the payoff value of volatile assets. Flight to collateral emphasizes a different channel, created by movements in collateral values. Even in the absence of flight to quality behavior (associated with movements in payoff values), we may still observe a relatively bigger price deterioration of bad quality assets due to a time-varying liquidity wedge and different leverage cycles.

Finally, the model also provides a testable forecasting result. At 1, the information volatilities of both  $E^G$  and  $E^B$  are zero. From this we might expect the margins for  $E^G$  and  $E^B$  to be zero at 1, or at least the same. However, contagion at  $D$  causes volatility in the prices of  $E^G$  and  $E^B$ , and thus positive margins at 1. The flight to collateral at  $D$  causes more price volatility for  $E^B$  than for  $E^G$ , and hence slightly higher margins at 1 for  $E^B$  than for  $E^G$ . Thus, the margins during normal times at 1 can predict which asset will suffer more during future flights to collateral during anxious times.

To sum up, different leverage cycles (i.e., different endogenous margin requirements) create flight to collateral and thus differential contagion during anxious times, which gives a rationale for Stylized Fact 2. Real world margins during normal times are about 10 percent for high-rated emerging market bonds and about 20 percent or more for low-rated emerging market bonds. Provided that the expected flow of future information across credit ratings is symmetric, these margins during normal times indicate that low-rated emerging market bonds will be the ones suffering during future flight to collateral episodes.

## V. Model II: Collateral General Equilibrium with Adverse Selection

In this section we focus on the issuance problem. For this, we extend the collateral model. Instead of taking the supply of  $E$  as fixed, we explicitly model the issuance choice of emerging market assets.

TABLE 13—SIMULATION 6, INCOMPLETE MARKETS WITH COLLATERAL, 3 ASSETS: PRICES

| Asset          | 1      | U       | D      | (U - D)/U<br>% | (1 - D)/1<br>% |
|----------------|--------|---------|--------|----------------|----------------|
| G              | 0.8699 | 0.8864  | 0.7726 | 12.8           | 11.2           |
| B              | 0.8458 | 0.8654  | 0.7298 | 15.7           | 13.7           |
| H              | 0.9311 | 0.9985  | 0.7332 | 26.5           | 21.2           |
| r <sub>s</sub> | 0.0000 | -0.0015 | 0.0005 |                |                |

TABLE 14—SIMULATION 6, INCOMPLETE MARKETS WITH COLLATERAL, 3 ASSETS: PRICE COMPONENTS, LIQUIDITY PREFERENCE, AND MARGINS

|        |    | 1      | U      | D      | (U - D)/p <sub>U</sub><br>% | (1 - D)/p <sub>1</sub><br>% |
|--------|----|--------|--------|--------|-----------------------------|-----------------------------|
| ω      |    | 0.0412 | 0.0409 | 0.2471 |                             |                             |
| Assets |    |        |        |        |                             |                             |
| G      | PV | 0.8394 | 0.8791 | 0.7327 | 16.5                        | 12.2                        |
|        | CV | 0.0306 | 0.0079 | 0.0396 | -3.7                        | -1.0                        |
|        | m  | 0.1119 | 0.7747 | 0.7410 |                             |                             |
| B      | PV | 0.8169 | 0.8636 | 0.7199 | 16.6                        | 11.5                        |
|        | CV | 0.0289 | 0.0020 | 0.0099 | -0.9                        | 2.2                         |
|        | m  | 0.1371 | 0.9423 | 0.9315 |                             |                             |

A. Model

*Emerging Countries.*—In each state,  $s \in S$ , each country  $ks$  chooses to issue assets. To simplify our calculation, we assume that each country has only one chance to issue assets and is not allowed to trade on secondary markets. We also assume that countries consume only at the period of issuance and at the end.<sup>29</sup> Each country  $ks$  has Bernoulli utility  $u^{ks}(x)$  for consumption of  $x$  units of the consumption good in state  $s$  and in states  $t \in S_T(s)$ , where  $S_T(s)$  is the set of terminal nodes that follow  $s$ . Utilities satisfy the usual assumptions discussed before. Country  $ks$  assigns subjective probability  $q_\alpha^{ks}$  to the transition from any state  $\alpha^*$  to  $\alpha$ . (Naturally  $q_s^{ks} = 1$ .) Letting  $\bar{q}_\alpha^{ks}$  be the product of all  $q_\beta^{ks}$  along the path from  $s$  to  $\alpha$ , the von Neumann–Morgenstern expected utility to country  $ks$  is

$$(14) \quad U^{ks} = \sum_{\alpha \in \{s\} \cup S_T(s)} \bar{q}_\alpha^{ks} (\delta^{ks})^{t(\alpha)-t(s)} u^{ks}(x_\alpha).$$

We denote the issuance at  $s$  of country  $ks$  by  $z_{ks}$ . Countries are endowed with the consumption good at each terminal node  $t \in S_T(s)$ . In the absence of any endowment, they need to issue debt in order to consume at  $s$ .

*Types and Symmetric Information.*—In each state  $s \in S$ , there are two types of countries, “good,”  $k = G$ , and “bad,”  $k = B$ , issuing assets in the primary market. Assets issued by different types differ in their deliveries; the good type always pays at least as much as the bad type:  $D_{\alpha G} \geq D_{\alpha B}, \forall \alpha \in S$ . We assume that the deliveries of countries of the same type are the same (even if they were issued at different states). Thus, all assets known to be good (bad) at  $s$  will trade for the same price  $p_{sG}$  ( $p_{sB}$ ), whether issued at  $s$  and trading on the primary market at  $s$ , or issued

<sup>29</sup> Adding intermediate consumption when countries are not allowed to trade or issue would not affect any of the results.

previously and trading on the secondary market at  $s$ . However, the prices  $p_{sG}$  and  $p_{sB}$  may or may not coincide.

Suppose provisionally that each country  $ks$  is obliged to issue all of its endowment of  $z_{ks}$  of its asset, giving total issuance  $\mathbf{z} = (z_{Bs}, z_{Gs})_{s \in S}$ . Suppose further that everyone knows the quality ( $k = G$  or  $k = B$ ) of each country, and therefore knows the payoffs of the assets they buy (conditional on the terminal state). Suppose finally that each issuing country is obliged to spend all its revenue from selling its asset on immediate consumption. A standard  $z$ -collateral equilibrium for this  $z$ -economy can then be defined exactly as before, with the obvious modification that the original owners of all the assets  $\mathbf{z} = (z_{Bs}, z_{Gs})_{s \in S}$  must sell them all for immediate consumption.

*Types and Asymmetric Information.*—Now suppose that there is asymmetric information: investors cannot observe a country's type and hence the type of credit they are trading. Furthermore, suppose that the countries are endowed with one unit of each asset and can choose how much to issue  $z_{ks}$ . Bad countries will have incentive to sell more because they know they will be required to deliver less in the terminal states.

*The Market as a Designer.*—At this point we face a problem: how can we make compatible the adverse selection problem arising from the asymmetric information with the perfect competition framework described in Model I? To attack this problem, we follow the modeling strategy used in Dubey and Geanakoplos (2002) to study insurance in a competitive framework. We apply their techniques to extend the collateral general equilibrium model of Section IV to encompass adverse selection and issuance rationing.

In each state  $s \in S$ , there are many different debt markets, each characterized by a quantity limit (which a seller in that market cannot exceed) and its associated market clearing price:

$$(15) \quad \vec{p}_s = \{(z_s, p_s(z_s)); z_s \in (0, 1], p_s \in \mathbb{R}_+\}.$$

The issuance-price schedule  $\vec{p}_s$  is taken as given, and emerging countries and investors decide in which of these debt markets to participate. We assume exclusivity, i.e., countries can issue (sell) in only one debt-quantity market at any given time. So they must choose a quantity  $z_s$  to sell and then take as given the corresponding market clearing price  $p_s(z_s)$ .

Given the price schedule  $\vec{p}_s$ , country  $ks$  decides consumption and issuance in order to maximize utility (14) subject to the budget set defined as

$$\begin{aligned} B^{ks}(\vec{p}_s) &= \{(\mathbf{x}, z) \in \mathbb{R}_+^{1+S_T(s)} \times \mathbb{R}_+ : \\ &x_s \leq \vec{p}_s(z)z, \\ &z \leq 1, \\ &\forall \alpha \in S_T(s): x_\alpha = e_\alpha^{ks} + (1 - z)D_{\alpha k}\}. \end{aligned}$$

Consumption at  $s$  has to be less than or equal to the income from issuance of quantity  $z$ . Issuance at  $s$  cannot exceed the total endowment of the asset  $k$  of 1 unit. Finally, consumption at each terminal node that follows  $s$  has to be less than or equal to the endowment of the consumption good plus the deliveries on the remaining asset that was not sold at  $s$ .

Investors who buy assets in market  $(z_s, \vec{p}_s(z_s))$  get a pro rata share of the deliveries of all assets sold in that market. If the proportion of the sales at  $z_s$  of the bad type exceeds the proportion of bad types in the economy, then the buyer at  $z_s$  gets an adverse selection. Investors are assumed to be rational and to have the correct expectation of deliveries from each market  $(z_s, \vec{p}_s(z_s))$ . Thus, if only one country type is choosing to sell at the quantity  $z_s$ , then it reveals its type, and from then on, its asset payoffs are known to be the corresponding type.

With this interpretation, there is room for *signalling* as well as *adverse selection* without destroying market anonymity. Countries may (falsely) signal more reliable deliveries by publicly committing to (small) quantity markets where the prices are high because the market expects only good types to sell there. The quantity limit characterizing each debt market is exogenous and the associated price is set endogenously as in any traditional competitive model. However, it may occur that in equilibrium only a few debt markets are active, even when all the markets are priced in equilibrium. In this sense, the active quantities are set endogenously as well, without the need of any contract designer. Market clearing and optimizing behavior are enough.

*Separating Collateral Equilibrium.*—A formal definition of equilibrium in this model is quite involved, because there are so many markets, and because the secondary market prices will depend on what is revealed in the primary markets. However, there is a shortcut to this problem. We say that an equilibrium is pooling if at any state  $s$  two countries of different types decide to sell the same amount, and hence participate in the same market. In contrast, an equilibrium is separating when different types,  $G_s$  and  $B_s$ , always issue different amounts in the same state. Dubey and Geanakoplos (2002) show that their model exhibits a unique refined separating equilibrium, and no equilibrium involving any pooling. Their techniques are still valid in the present model to show the existence of a separating equilibrium.<sup>30</sup> A formal definition of a separating equilibrium is simpler.

A *Separating Collateral Equilibrium*  $((\vec{p}, r), (x^i, y^i, \phi^i)_{i \in I}, (x_{G_s}, z_{G_s}, x_{B_s}, z_{B_s})_{s \in S})$  satisfies the following:

1.  $((p_{G_s}, p_{B_s}, r_s)_{s \in S}, (x^i, y^i, \phi^i)_{i \in I})$  is a standard  $z$ -collateral equilibrium, where  $z = (z_{G_s}, z_{B_s})_{s \in S}$ ;
2.  $(p_{G_s}, p_{B_s})_{s \in S} = (\vec{p}_s(z_{G_s}), \vec{p}_s(z_{B_s}))_{s \in S}$  and for all  $s \in S$ ;
3.  $(x_{G_s}, z_{G_s})$  is optimal for country  $G_s$  in  $B^{G_s}(\vec{p}_s)$  and  $(x_{B_s}, z_{B_s})$  is optimal for country  $B_s$  in  $B^{B_s}(\vec{p}_s)$ ;
4.  $z_{G_s} < z_{B_s}$ .<sup>31</sup>

Finally, let us stress why it is so important that the model exhibit a separating equilibrium from a computational point of view. In general, equilibrium would have forced us to solve for prices for all possible quantity limits, and to distinguish assets sold later by how much of them were originally issued. This is an infinite dimensional problem. In a separating equilibrium, we

<sup>30</sup> Whether there are other equilibria in this model is an open question.

<sup>31</sup> The definition of equilibrium requires prices  $\vec{p}_s(z_s)$  for those markets  $z_s$  that are not active to be determined as well. For  $z_s < z_{G_s}$ ,  $\vec{p}_s(z_s)$  is determined so that the good type is indifferent between issuing  $z_s$  and  $z_{G_s}$ . For  $z_s \geq z_{G_s}$ ,  $\vec{p}_s(z_s)$  is such that the bad type is indifferent between  $z_s$  and  $z_{B_s}$ . This separating equilibrium is robust to refinements, as shown in Dubey and Geanakoplos (2002).

need only keep track of good and bad asset prices,  $p_{G_s}$  and  $p_{B_s}$ , and good and bad issuance levels,  $z_{G_s}$  and  $z_{B_s}$ . This reduces the problem to a finite set of variables, as we had before.

### B. Leverage Cycles, Adverse Selection, and Issuance Rationing

Simulation 7 solves the equilibrium for the same parameters as before. The new parameters are the ones describing countries. Utilities for the countries are quadratic:  $U^{ks} = (x_s - \beta x_s^2) + \sum_{s' \in T(s)} \bar{q}_\alpha^{ks} (\delta^{ks})^{t(s)} (x_{s'} - \beta x_{s'}^2)$  with  $\beta = 1/370$ . Endowments and beliefs for the countries are the same as those of the optimistic investors, so  $e_{s' \in T(s)}^{ks} = 20$ ,  $q^{ks} = 0.9$ . Tables 15 and 16 present the results. The price behavior described in Simulation 6 is still present here: there is contagion and flight to collateral. Portfolio and consumption effects are present; hence both emerging market asset prices fall from  $U$  to  $D$  and from 1 to  $D$ . Moreover, different leverage cycles create an increase in the spread between types  $G$  and  $B$  from 1 to  $D$ .

The new element in this simulation comes from the supply side. At  $D$  there is a drop in issuance and, more importantly, a more severe drop for the good type. The bad type issuance goes from  $z_{B1} = 1$  to  $z_{BD} = 0.75$ , whereas the good type issuance goes from  $z_{G1} = 0.8$  all the way to  $z_{GD} = 0.08$ . The gap in issuance between  $U$  and  $D$  is also bigger for the good type than for the bad type. Now, adverse selection plays the leading role.<sup>32</sup>

It is not surprising that with contagion and the corresponding fall in prices, equilibrium issuance falls as well. The interesting thing is that flight to collateral combined with informational asymmetries generates issuance rationing: the fall in price of the good type is less, yet its drop in issuance is much more. The greater the spread between types, the more drastic is the drop in good quality issuance.

The explanation is that the bigger price spread between types requires a smaller good type issuance for a separating equilibrium to exist. Unless the good issuance levels become onerously low, bad types would be more tempted by the bigger price spread to mimic good types and sell at the high price  $p_{G_s}$ . The good types are able to separate themselves by choosing low enough  $z_{G_s}$  since it is more costly for the bad type to rely on the payoff of its own asset for final consumption than it is for the good type.

In standard models of adverse selection, incentive compatibility constraints play a central role. In the present model, with adverse selection embedded in a general equilibrium framework, the presence of a price-issuance schedule and utility maximization subject to budget constraints is enough.

In a world with no informational noise, spillovers from other markets may even help good issuance relative to bad issuance. However, if to market incompleteness, investor disagreement, and heterogeneous endogenous margin requirements we add some degree of informational noise between countries and investors, good quality issuance suffers more. In other words, contagion combined with flight to collateral and informational asymmetries creates issuance rationing.<sup>33</sup> This result solves our third problem, and in particular rationalizes Stylized Fact 3: high-rated issuance falls more than low-rated issuance during closures despite the fact that high-rated spreads increase less than low-rated spreads.

<sup>32</sup> As in Section IV, the simulation is robust to other choices of parameters.

<sup>33</sup> One may wonder at the role of credit agencies as information revealing devices. To make our explanation consistent with the existence of rating agencies, we need to assume that credit agencies do not know anything more than can be inferred from price, and that in effect they just follow the market.



TABLE 15—SIMULATION 7, INCOMPLETE MARKETS WITH COLLATERAL AND ADVERSE SELECTION: PRICES

| Asset | 1      | $U$    | $D$    | $(U - D)/U$<br>% | $(1 - D)/1$<br>% |
|-------|--------|--------|--------|------------------|------------------|
| $G$   | 0.8149 | 0.8409 | 0.6957 | 17.3             | 14.6             |
| $B$   | 0.7807 | 0.8117 | 0.6385 | 21.3             | 18.2             |
| $H$   | 0.8849 | 0.9967 | 0.6326 | 36.5             | 28.5             |
| $r_s$ | 0.0000 | 0.0000 | 0.0000 |                  |                  |

TABLE 16—SIMULATION 7, INCOMPLETE MARKETS WITH COLLATERAL AND ADVERSE SELECTION: ISSUANCE

| Type | 1      | $U$    | $D$    | $(U - D)/U$<br>% | $(1 - D)/1$<br>% |
|------|--------|--------|--------|------------------|------------------|
| $G$  | 0.8018 | 0.8524 | 0.0808 | 90               | 89.9             |
| $B$  | 1.0000 | 1.0000 | 0.7500 | 25               | 25               |

## REFERENCES

- ▶ **Agenor, Pierre-Richard, and Joshua Aizenman.** 1998. "Contagion and Volatility with Imperfect Credit Markets." *International Monetary Fund Staff Papers*, 45(2): 207–35.
- ▶ **Aghion, Philippe, Philippe Bacchetta, and Abhijit Banerjee.** 2006. "A Corporate Balance-Sheet Approach to Currency Crises." *Journal of Economic Theory*, 119(1): 6–30.
- ▶ **Alfaro, Laura, and Fabio Kanczuk.** 2005. "Sovereign Debt as a Contingent Claim: A Quantitative Approach." *Journal of International Economics*, 65(2): 297–314.
- ▶ **Bernanke, Ben, Mark Gertler and Simon Gilchrist.** 1996. "The Financial Accelerator and the Flight to Quality." *Review of Economics and Statistics*, 78(1): 1–15.
- ▶ **Bisin, Alberto, and Piero Gottardi.** 2006. "Efficient Competitive Equilibria with Adverse Selection." *Journal of Political Economy*, 114(3): 485–516.
- ▶ **Brunnermeier, Markus, and Lasse Pedersen.** Forthcoming. "Market Liquidity and Funding Liquidity." *Review of Financial Studies*.
- ▶ **Bulow, Jeremy, and Kenneth Rogoff.** 1989. "A Constant Recontracting Model of Sovereign Debt." *Journal of Political Economy*, 97(1): 155–78.
- ▶ **Caballero, Ricardo J., and Arvind Krishnamurthy.** 2001. "International and Domestic Collateral Constraints in a Model of Emerging Market Crises." *Journal of Monetary Economics*, 48(3): 513–48.
- ▶ **Calvo, Guillermo A.** 1998. "Capital Flows and Capital-Market Crises: The Simple Economics of Sudden Stops." *Journal of Applied Economics*, 1(1): 35–54.
- ▶ **Calvo Guillermo A.** 1999. "Contagion in Emerging Markets: When Wall Street Is a Carrier." *Proceedings from the International Economic Association Congress*, Vol. 3. Buenos Aires, Argentina.
- ▶ **Calvo, Guillermo A., Alejandro Izquierdo, and Luis-Fernando Mejía.** 2004. "On the Empirics of Sudden Stops: The Relevance of Balance-Sheet Effects." National Bureau of Economic Research Working Paper 10520.
- ▶ **Calvo, Guillermo A., and Enrique G. Mendoza.** 2000. "Contagion, Globalization, and the Volatility of Capital Flows." In *Capital Flows and the Emerging Economies: Theory, Evidence, and Controversies*, ed. Sebastian Edwards, 15–41. Chicago: University of Chicago Press.
- ▶ **Catão Luis, Ana Fostel, and Sandeep Kapur.** Forthcoming. "Persistent Gaps and Default Traps." *Journal of Development Economics*.
- ▶ **Chang, Roberto, and Andres Velasco.** 2001. "A Model of Financial Crises in Emerging Markets." *Quarterly Journal of Economics*, 116(2): 489–517.
- ▶ **Cipriani Marco, and Antonio Guarino.** Forthcoming. "Informational Cascades and Contagion in Financial Markets." *The B.E. Journal of Theoretical Economics*.
- ▶ **Corsetti, Giancarlo, Paolo Pesenti, and Nouriel Roubini.** 1999. "Paper Tigers? A Model of the Asian Crisis." *European Economic Review*, 43(7): 1211–36.
- ▶ **Dubey, Pradeep, and John Geanakoplos.** 2002. "Competitive Pooling: Rothschild–Stiglitz Reconsidered." *Quarterly Journal of Economics*, 117(4): 1529–70.
- ▶ **Eaton, Jonathan.** 1996. "Sovereign Debt, Reputation, and Credit Terms." *International Journal of Finance and Economics*, 1(1): 25–35.

- Fostel, Ana.** 2005. "A Supply Story: Emerging Markets, High Yield and Primary Market Closures." PhD diss. Yale University.
- ▶ **Fostel, Ana, and John Geanakoplos.** 2008. "Collateral Restrictions and Liquidity under-Supply: A Simple Model." *Economic Theory*, 35(3): 441–67.
- Fostel, Ana, and Graciela Kaminsky.** Forthcoming. "Latin America's Access to International Capital Markets: Good Behavior or Global Liquidity?" In *Current Account and External Financing*, ed. Kevin Cowan, Sebastian Edwards, and Rodrigo Valdes. Santiago: Central Bank of Chile.
- ▶ **Gale, Douglas.** 1992. "A Walrasian Theory of Markets with Adverse Selection." *Review of Economic Studies*, 59(2): 229–55.
- Geanakoplos, John.** 1997. "Promises, Promises." In *The Economy as an Evolving Complex System II*, ed. W. Brian Arthur, Steven Durlauf, and David Lane, 285–320. Reading, MA: Addison-Wesley.
- Geanakoplos, John.** 2003. "Liquidity, Default, and Crashes: Endogenous Contracts in General Equilibrium." In *Advances in Economics and Econometrics: Theory and Applications, Eighth World Conference*, Vol. 2, 170–205. Econometric Society Monographs.
- Geanakoplos, John, and William Zame.** 1998. "Default, Collateral, and Crashes." Unpublished.
- Goldfajn, Ilan, and Rodrigo Valdes.** 1997. "Capital Flows and the Twin Cases: The Role of Liquidity." International Monetary Fund Working Paper 97/87.
- Gonzales, Martin, and Eduardo Levy Yeyati.** 2005. "Global Factors and Emerging Market Spreads". Inter-American Development Bank Working Paper 552.
- International Monetary Fund.** 2003. *Global Financial Stability Report*. March 2003 and September 2003. Washington, DC: International Monetary Fund Publications.
- Kaminsky, Graciela L., and Carmen M. Reinhart.** 1999. "The Twin Crises: The Causes of Banking and Balance-of-Payments Problems." *American Economic Review*, 89(3): 473–500.
- ▶ **King, Mervyn A., and Sushil Wadhvani.** 1990. "Transmission of Volatility between Stock Markets." *Review of Financial Studies*, 3(1): 5–33.
- ▶ **Kiyotaki, Nobuhiro, and John Moore.** 1997. "Credit Cycles." *Journal of Political Economy*, 105(2): 211–48.
- ▶ **Kodres, Laura E., and Matthew Pritsker.** 2002. "A Rational Expectations Model of Financial Contagion." *Journal of Finance*, 57(2): 768–99.
- ▶ **Krugman, Paul.** 1979. "A Model of Balance-of-Payments Crises." *Journal of Money, Credit, and Banking*, 11(3): 311–25.
- Krugman, Paul.** 1999. "Balance Sheets, the Transfer Problem, and Financial Crises." In *International Finance and Financial Crises: Essays in Honor of Robert P. Flood, Jr.*, ed. Peter Isard, Assaf Razin, and Andrew K. Rose, 31–44. Boston: Kluwer Academic.
- ▶ **Kyle, Albert S., and Wei Xiong.** 2001. "Contagion as a Wealth Effect." *Journal of Finance*, 56(4): 1401–40.
- ▶ **Martin, Philippe, and Hélène Rey.** 2006. "Globalization and Emerging Markets: With or Without Crash?" *American Economic Review*, 96(5): 1631–51.
- Mendoza, Enrique.** 2004. "Sudden Stops in an Equilibrium Business Cycle Model with Credit Constraints: A Fisherian Deflation of Tobin's Q." Unpublished.
- Obstfeld, Maurice.** 1994. "The Logic of Currency Crises." *Cahiers Economiques et Monétaires*, 43: 189–213.
- Pavlova, Anna, and Roberto Rigobon.** Forthcoming. "The Role of Portfolio Constraints in the International Propagation of Shocks." *Review of Economic Studies*.
- ▶ **Rothschild, Michael, and Joseph E. Stiglitz.** 1976. "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information." *Quarterly Journal of Economics*, 90(4): 630–49.
- Rustichini, Aldo, and Paolo Siconolfi.** Forthcoming. "General equilibrium in economies with adverse selection." *Economic Theory*.
- ▶ **Schneider, Martin, and Aaron Tornell.** 2004. "Balance Sheet Effects, Bailout Guarantees and Financial Crises." *Review of Economic Studies*, 71(3): 883–913.
- Vayanos, Dimitri.** 2004. "Flight to Quality, Flight to Liquidity, and the Pricing of Risk." Unpublished.